Spring 2017 McNabb GDCTM Contest
Calculus

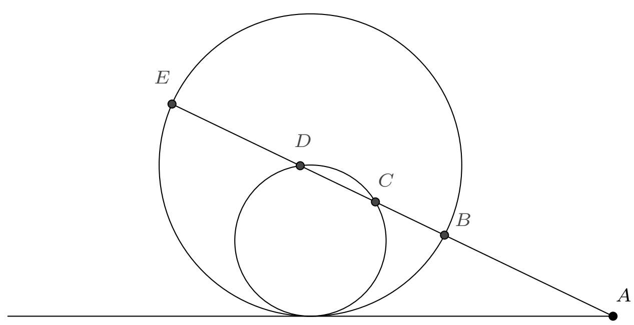
NO Calculators Allowed

Assume all variables are real unless otherwise stated in the problem.

1. Four cards are randomly removed from a standard deck of 52 playing cards. Then one of the four cards that were removed is chosen at random. What is the probability that card is an ace?

2. A coach takes orders for seven of his players. Each player chooses exactly one of these items: hot dogs, nachos, chicken tenders, salad. How may different *orders* can occur? That is, do not take into account which specific player gets a given item. (When the coach arrives each player is happy!)

3. Circle R and smaller circle S are internally tangent to each other at point P and both externally tangent to line n, also at point P. A second line m cuts circle R at points B and E, circle S at points C and D, and line n at point A, so that points A, B, C, D, E occur in that order on m. If $AB = 3$, and $BC = CD = 1$, find DE.



4. Let f be a function continuous on the interval $[0, 10]$. Suppose that the average value of f on the interval $[0, 10]$ is six and the average value of f on the interval $[6, 10]$ is four. Find the average value of f on the interval $[0, 6]$.

5. Find the value of the constant a which minimizes the value of

$$\int_{-1}^{1} (x^3 - ax)^2 \, dx$$

6. For which certain values of y must solutions of the differential equation

$$\frac{dy}{dx} = y(y-1)(y-2)$$

have a point of inflection (assuming these solutions pass through a point with such a y coordinate)?

7. Let a, b, c, and d, be four distinct integers whose product is 1024. Find the least possible value of their sum.

8. Andy the ant starts at a certain vertex of a cube and randomly chooses which of the three edges that meet that vertex to travel on to the next vertex. When Andy reaches that next vertex he again randomly chooses which of the three edges to take. This process continues on and on. What is the probability that after Andy's fourth choice of edge is traversed, he has returned to the vertex from which he began his random walk?

9. Rectangle $ABCD$ is a rectangular billiard table with $AD = 1$ and $AB = 3$. A ball is at point P on side CD with $DP = 1$. A player aims the ball at point Q on side BC so that after three caroms (bounces) the ball will be headed back to where it started. Find CQ.

10. A solid is constructed on a square of side length one so that cross-sections perpendicular to one of the square's diagonals are semicircles. Find the volume of this solid.

11. Find the four-dimensional volume of the hypersphere of radius one given by the relation

$$x^2 + y^2 + z^2 + w^2 \leq 1$$

12. Evaluate

$$\int_5^\infty \frac{1}{\lfloor x \rfloor} - \frac{1}{\lfloor x+1 \rfloor}\, dx$$

13. Find the coefficient of x^{10} in the Maclaurin series of

$$f(x) = \frac{x}{2x^2 + 3x + 1}$$

14. Evaluate

$$\int_0^{\pi/3} 8\cos^4 x - 8\cos^2 x + 1\, dx$$

15. Six children are on a merry-go-round with six seats, all facing the same way. They decide to change how they are seated so that each child will have a different child in front of them (all still facing the same way). In how many ways can this be done?