Spring 2017 McNabb GDCTM Contest
PreCalculus

NO Calculators Allowed

1. True or False:
$$\frac{\sqrt{5}+\sqrt{7}}{2} > \sqrt{6}$$

2. For what real value(s) of the constant k does the equation
$$\frac{x}{x+3} - \frac{1}{x-3} = \frac{kx}{x^2-9}$$
have exactly one real valued solution?

3. What is the remainder when 123456654321 is divided by seven?

4. Simplify $\sqrt[3]{970299}$.

5. Find one ordered pair of positive integers (m, n) so that
$$21m - 34n = 1$$

6. Write $\cos 15^\circ$ in simplified radical form.

7. Find one complex number z in the form $a + bi$ where a and b are real with $a \neq 0$ and $b \neq 0$, such that
$$z^6 + 6z^5 + 15z^4 + 20z^3 + 15z^2 + 6z = 0$$

8. In a city of $100,000$ souls, 400 carry gene G. A test for gene G is 99% accurate in both directions, meaning that if a person has gene G, the test will be positive 99% of the time and that if a person does not have the gene G the test will be positive 1% of the time. A randomly selected resident of the city tests positive for gene G. What is the probability that this person actually has gene G? Answer as a fraction. Warning: you may be surprised by the answer!

9. Solve for x:
$$\begin{vmatrix} x^2 & x & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$$

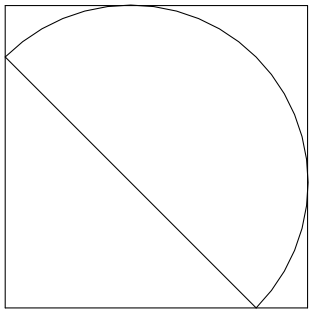
10. Find the number of ways to mail ten distinct books if two packages contain three books each and one package contains four books?

11. A coach takes orders for seven of his players. Each player chooses exactly one of these items: hot dogs, nachos, chicken tenders, salad. How may different *orders* can occur? That is, do not take into account which specific player gets a given item. (When the coach arrives each player is happy!)

12. Starting at the origin take unit steps to the right or left with equal probability. Stop when you reach the points -3 or 3. What is the average number of steps you will take, when you repeat this experiment over and over?

13. A regular hexagon $ABCDEF$ has center G and side lengths 2. The hexagon is divided into six equilateral triangles in the usual way (such as $\triangle GAB$, etc...). The centers of mass of these six equilateral triangles form hexagon $HIJKLM$. Find the ratio of the area of $HIJKLM$ to the area of $ABCDEF$.

14. A semicircle is inscribed in a square as shown. This means that both endpoints of the diameter of the semicircle lie on the square, and at the other two points of contact, the side of the square is tangent to the semicircle. Find the ratio of the area of the semicircle to the area of the square.



15. Six children are on a merry-go-round with six seats, all facing the same way. They decide to change how they are seated so that each child will have a different child in front of them (all still facing the same way). In how many ways can this be done?