

SPRING 2017 MCNABB GDCTM CONTEST  
ALGEBRA TWO

NO Calculators Allowed

1. Which is larger:  $7^{14}$  or  $14^7$ ?

2. Find just one ordered pair of integers  $(m, n)$  so that

$$m^2 + mn + n^2 = 39$$

3. Write 621 as the sum of three perfect squares.

4. Let  $(a, b)$  be fixed positive real numbers. Find the area of the parallelogram formed by the four lines

$$y = ax - b$$

$$y = ax + b$$

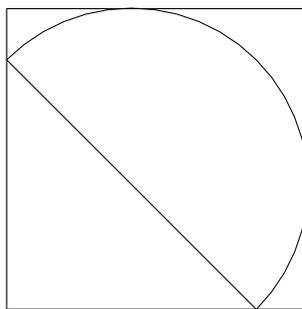
$$y = -ax - b$$

$$y = -ax + b$$

in terms of  $a$  and  $b$ .

5. Find the minimum possible value of  $ax + by + cz$  if  $\{a, b, c\} = \{4, 5, 6\}$  and  $\{x, y, z\} = \{7, 8, 9\}$ .

6. A semicircle is inscribed in a square as shown. This means that both endpoints of the diameter of the semicircle lie on the square, and at the other two points of contact, the side of the square is tangent to the semicircle. Find the ratio of the area of the semicircle to the area of the square.



7. Let  $x$ ,  $y$ , and  $z$  be the solutions of the system

$$\begin{aligned}x + 2y - z &= -1 \\2x - y + z &= 9 \\x + 3y + 3z &= 6\end{aligned}$$

Find the value of  $10x + 2y + z$ .

8. Four cards are randomly removed from a standard deck of 52 playing cards. Then one of the four cards that were removed is chosen at random. What is the probability that card is an ace?

9. In how many ways can two subsets of

$$S = \{a, b, c, d, e, f\}$$

be chosen so that their union is  $S$  and their intersection contains three letters? The order of the subsets is not material.

10. Find the coefficient of  $x^6$  when

$$(x^3 - 6x^2 + 11x - 6)^3$$

is expanded and simplified.

11. Let  $\angle A = 60^\circ$ . A circle of radius 1 is dropped into this angle so it fits as snugly as possible (closest to point  $A$ ). What is the radius of the circle that will fit snugly on top of the first circle and be tangent to both sides of the angle?
12. In a city of 100,000 souls, 400 carry gene  $G$ . A test for gene  $G$  is 99% accurate in both directions, meaning that if a person has gene  $G$ , the test will be positive 99% of the time and that if a person does not have the gene  $G$  the test will be positive 1% of the time. A randomly selected resident of the city tests positive for gene  $G$ . What is the probability that this person actually has gene  $G$ ? Answer as a fraction. Warning: you may be surprised by the answer!
13. Let  $r$ ,  $s$ , and  $t$  be the roots of the polynomial  $x^3 - 4x^2 - 11x + 30$ . Find the value of  $r^2 + s^2 + t^2$ .
14. It is known that every isosceles trapezoid can be inscribed in a circle. Find the radius of that circle if the side lengths of the isosceles trapezoid are 3, 5, 9, and 5.
15. Let  $a$ ,  $b$ ,  $c$ , and  $d$  be real numbers satisfying

$$\begin{aligned}a^2 + b^2 &= 37 \\c^2 + d^2 &= 26 \\ac - bd &= 11\end{aligned}$$

Find the largest possible value of  $ad + bc$ .