Spring 2017 McNabb GDCTM Contest
Geometry

NO Calculators Allowed

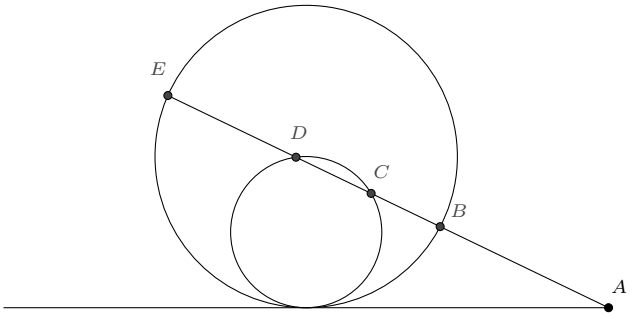
1. Find the volume of a sphere of radius 3.
2. In triangle ABC with $\angle C$ right, a square built on side AC has area 64 while a square built on side AB has area 100. Find the length of side BC.
3. Billy sells \$3471 worth of chocolate boxes. He sells two kinds of boxes, a milk chocolate mix at \$15 per box, and a dark chocolate assortment at \$14 per box. If he sells a total of 236 boxes, how many boxes of the dark chocolate assortment does he sell?
4. Let
$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{g}{h} = \frac{4}{5}$$
then what is the value of
$$\frac{a^2 + c^2 + e^2 + g^2}{b^2 + d^2 + f^2 + h^2}$$
?
5. A circular pond with volume 36π cubic feet and depth 4 feet is having a circular walkway built around it. The walkway should be 4 feet wide and be sunk 2 feet into the ground. What volume of concrete is needed to build the walkway? Answer in cubic feet.
6. Given rectangle $ABCD$ let E and F be the midpoints of sides AB and CD respectively. Draw a circle with diameter EF of length 10. If the ratio of the area of the circle to the area of the rectangle is $\pi/8$, find the perimeter of the rectangle.
7. A quarter-circle is inscribed in a $30^\circ - 60^\circ - 90^\circ$ triangle, with its center at the vertex of the right angle. If the area of the quarter circle is 36π, what is the area of the triangle?
8. Convex hexagon $ABCDEF$ has
$$EF = FA = AB = BC = 2$$
$$\angle F = \angle A = \angle B = 150^\circ$$
$$\angle E = \angle D = \angle C = 90^\circ$$
What is the area of this hexagon?
9. In $\triangle ABC$, $AB = 16$, $BC = 5$, and $\angle B = 120$ degrees. Find AC.
10. Let (a, b) be fixed positive real numbers. Find the area of the parallelogram formed by the four lines
$$y = ax - b$$
$$y = ax + b$$
$$y = -ax - b$$
$$y = -ax + b$$

in terms of a and b.

11. A right triangle has all integer side lengths, the smallest of which is eleven. Find the area of the triangle.

12. Rectangle $ABCD$ is a rectangular billiard table with $AD = 1$ and $AB = 3$. A ball is at point P on side CD with $DP = 1$. A player aims the ball at point Q on side BC so that after three carooms (bounces) the ball will be headed back to where it started. Find CQ.

13. Circle R and smaller circle S are internally tangent to each other at point P and both externally tangent to line n, also at point P. A second line m cuts circle R at points B and E, circle S at points C and D, and line n at point A, so that points A, B, C, D, E occur in that order on m. If $AB = 3$, and $BC = CD = 1$, find DE.



14. A circle of area 36π is inscribed in square $ABCD$. Side AB is extended past B to point E and side AD is extended past D to point F in such a way that C lies on EF. Find the minimum possible area of $\triangle EAF$.

15. In rectangle $ABCD$, let F be the midpoint of AB, and points E and G be the midpoints of AF and FB respectively. Draw diagonal AC and segments DE, DF and DG intersecting AC at points H, I, J respectively. Find the ratio HI/IJ.