Spring 2017 McNabb GDCTM Contest
Algebra One

NO Calculators Allowed

1. Hezy paid $19.04 for a shirt that was marked 15% off. What was the original price of the shirt?

2. Let $x = 0.\overline{57}$ and $y = 0.\overline{03}$. Write $x + y$ in simplified fraction form.

3. Define the function
$$g(a, b, c) = \left(\frac{a^3 + b^3 + c^3}{a + b + c} \right)^2$$
Find the value of $g(3, 4, 5)$.

4. Let x, y, and z be the solutions of the system
$$x + 2y - z = -1$$
$$2x - y + z = 9$$
$$x + 3y + 3z = 6$$
Find the value of $10x + 2y + z$.

5. A circular pond with volume 36π cubic feet and depth 4 feet is having a circular walkway built around it. The walkway should be 4 feet wide and be sunk 2 feet into the ground. What volume of concrete is needed to build the walkway? Answer in cubic feet.

6. True or False:
$$\frac{\sqrt{5} + \sqrt{7}}{2} > \sqrt{6}$$

7. Billy sells $3471 worth of chocolate boxes. He sells two kinds of boxes, a milk chocolate mix at $15 per box, and a dark chocolate assortment at $14 per box. If he sells a total of 236 boxes, how many boxes of the dark chocolate assortment does he sell?

8. How many positive integers less than five-hundred are relatively prime to either 16 or 27 but not both?

9. Find distinct positive integers (p, q, r) so that $p < q < r$ and
$$\frac{7}{11} = \frac{1}{p} + \frac{1}{q} + \frac{1}{r}$$
Write your answer as (p, q, r)

10. Eve is standing at the exact center of an orchard, whose trees are evenly spaced in 16 rows, with 16 trees in each row, making a square array. How many of the trees that form the boundary of the orchard can she see? Treat the trees as skinny poles.

11. Three consecutive integers have the property that the cube of their sum minus nine times the sum of their cubes equals -918. What is the smallest of these integers?

12. A quadratic function p satisfies $p(7) = -5$, $p(8) = 4$, and $p(9) = -7$. Find the value of $p(6)$.

13. Store-owner Josie wants to price items in dollars and cents at her store between $10.00 and $20.00 inclusive so that even after a 15% discount they are still exact. For instance, she would not want to price an item at $11.25 because after a 15% discount, the price would be $9.5625, which is not of the form $ab.cd$. How many initial prices meet Josie's requirements?

14. Let a, b, c, and d be real numbers satisfying

$$a^2 + b^2 = 37$$
$$c^2 + d^2 = 26$$
$$ac - bd = 11$$

Find the largest possible value of $ad + bc$.

15. Napoleon's army is on retreat from Russia back to France forming a column 20 miles long. Napoleon on his horse at the back of the column proceeds to the front of the column, then returns to the back, taking 7 hours altogether. If his army marches at 3 mph, how fast does Napoleon's horse trot?