Spring 2016 McNabb GDCTM Contest
Pre-Algebra

NO Calculators Allowed

1. What percent of 45 is 36?

2. Cindy is 3 miles away from home. If she walks at a rate of 4 miles per hour, in how many minutes will she arrive at home?

3. How many edges does a prism with hexagonal bases have?

4. In a certain triangle the base is doubled and the height is tripled. What is the ratio of the area of the new triangle to the area of the orginal triangle?

5. Music streaming company Ossify charges a flat monthly fee of \$8 but charges 10 cents for each hour of listening over 50 hours for a given month. On the other hand, music streaming company Panaplex charges a flat monthly fee of \$6 but charges 14 cents for each hour of listening over 40 hours for a given month. How many total hours of listening would be required per month to make the total charges for that month from these companies turn out to be the same?

6. How many even postitive integers are factors of $3^5 - 1$?

7. Three times the complement of what angle is equal to the supplement of that angle?

8. Hezy, Zeke, and Elias are running around a track in the same direction. Each of them runs at their own constant pace. Hezy is the fastest and passes Elias every 8 minutes. Meanwhile, Elias passes Zeke every 12 minutes. So how many seconds elapse between times Hezy passes Zeke?

9. Admission to a zoo was \$ 20 per person when it was reduced to a new, lower rate. This caused the number of customers per day to increase by 40%. This in turn caused the amount collected by the zoo per day from admissions to increase by 12%. What is this new lower admission fee per person?

10. In how many ways can the letters in DALLAS be arranged so that neither the A's nor the L's are next to each other?

11. A group of 7th and 8th graders took the same math contest. The average score of all these students was 30. The average 7th grade score was 28 while the average 8th grade score was 33. What is the ratio of the number of 7th graders to the number of 8th graders?

12. Find the 17th decimal place in the decimal expansion of the fraction 17/2200.

13. Find the smallest value of the positive integer n so that the sum

$$1 + 2 + 3 + 4 + 5 + \cdots + n$$

is divisible by 100.

14. Today my son is 1/5 of my age. Two years ago he was 1/7 of my age. In how many years from today will he be 1/3 of my age?

15. Let $P = \{1, 4, 9, 16, 25, ...\}$ be the set of the squares of the positive integers. For how many elements p of P is $p + 144$ also an element of P?

Spring 2016 McNabb GDCTM Contest
Algebra One

NO Calculators Allowed

1. Cindy is 3 miles away from home. If she walks at a rate of 4 miles per hour, in how many minutes will she arrive at home?

2. Music streaming company Ossify charges a flat monthly fee of $8 but charges 10 cents for each hour of listening over 50 hours for a given month. On the other hand, music streaming company Panaplex charges a flat monthly fee of $6 but charges 14 cents for each hour of listening over 40 hours for a given month. How many total hours of listening would be required per month to make the total charges for that month from these companies turn out to be the same?

3. Find all solutions of
$$20x^2 + 33x - 27 = 0$$

4. Three times the complement of what angle is equal to the supplement of that angle?

5. Hezy, Zeke, and Elias are running around a track in the same direction. Each of them runs at their own constant pace. Hezy is the fastest and passes Elias every 8 minutes. Meanwhile, Elias passes Zeke every 12 minutes. So how many seconds elapse between times Hezy passes Zeke?

6. Admission to a zoo was $ 20 per person when it was reduced to a new, lower rate. This caused the number of customers per day to increase by 40%. This in turn caused the amount collected by the zoo per day from admissions to increase by 12%. What is this new lower admission fee per person?

7. A group of 7th and 8th graders took the same math contest. The average score of all these students was 30. The average 7th grade score was 28 while the average 8th grade score was 33. What is the ratio of the number of 7th graders to the number of 8th graders?

8. Today my son is 1/5 of my age. Two years ago he was 1/7 of my age. In how many years from today will he be 1/3 of my age?

9. Find one ordered triple of distinct positive integers (p, q, r) so that $p < q < r$ and
$$\frac{3}{10} = \frac{1}{p} + \frac{1}{q} + \frac{1}{r}$$
Write your answer as (p, q, r)

10. If a, b, and c are positive integers satisfying $abc = 1560$, find the least possible value of $a + b + c$.

11. Find the least value of x that satisfies
$$|5x - 70| \leq |4x - 200|.$$

12. Find the area of the parallelogram formed by the four lines
$$\begin{aligned} y &= 3x - 7 \\ y &= 3x + 7 \\ y &= 7x - 3 \\ y &= 7x + 3 \end{aligned}$$

13. I have only dimes and quarters in my pocket. There are b coins in all, and all together they are worth c cents. In terms of b and c, how many quarters do I have in my pocket?

14. When the cubic polynomial $x^3 - x^2 + kx - 2$ is divided by $x - 3$ the remainder is k. Find the value of the constant k.

15. Let $P = \{1, 4, 9, 16, 25, ...\}$ be the set of the squares of the positive integers. For how many elements p of P is $p + 144$ also an element of P?

Spring 2016 McNabb GDCTM Contest
Geometry

NO Calculators Allowed

1. How many edges does a prism with hexagonal bases have?
2. Find the ratio of the square of the circumference of a circle to the area of that same circle.
3. Find the value of k for which the point $(3k-1, k)$ lies on the line $7x - 3y = 2$.
4. Three times the complement of what angle is equal to the supplement of that angle?
5. In a certain triangle the base is doubled and the height is tripled. What is the ratio of the area of the new triangle to the area of the orginal triangle?
6. The front face of a rectangular box has area 72. Its left face has area 48 while its top face has area 96. Find the volume of the box.
7. Find the area of a triangle with sides of length 9, 10, and 11.
8. Find the area of the parallelogram formed by the four lines

$$\begin{aligned} y &= 3x - 7 \\ y &= 3x + 7 \\ y &= 7x - 3 \\ y &= 7x + 3 \end{aligned}$$

9. Find the coordinates of the center of the circumcircle of the triangle whose vertices are given by: $(2,0), (0,2)$, and $(10,0)$.
10. In $\triangle ABC$, the bisector of $\angle A$ meets side BC at point D. Find the ratio of the area of $\triangle ABD$ to the area of $\triangle ADC$ if $AB = 13$ and $AC = 17$.
11. Let points A, B, C, and D lie evenly spaced on a line in that order. On BC as base an equilateral triangle BCP is drawn. If $AB = 12$, determine AP.
12. Two congruent circles have a common external tangent of length 20 and a common internal tangent of length 18. What is the common radius of the two circles?

13. Two circles intersect at points A and B. The tangents to the two circles at point A meet at right angles. The radius of the smaller circle is 8 and the radius of the larger circle is 15. Find the length of the AB.

14. A convex pentagon has side lengths in cyclic order as: $17, 6, 13, 26$, and 4. The sides of lengths 26 and 6 are parallel, and the sides of lengths 26 and 4 are perpendicular. What is the area of the pentagon?

15. In $\triangle ABC$, $AB = AC$, and P and Q are the midpoints respectively of AB and AC. Extend BC to point D so that $CD = BC$. Let PD meet AC at point R. Find the ratio of QR to AC.

Spring 2016 McNabb GDCTM Contest
Algebra II

NO Calculators Allowed

1. A thirteen foot tree is growing at a rate of three feet per year while a forty-one foot tree is growing at a rate of two feet per year. In how many years will the two trees be the same height?

2. The front face of a rectangular box has area 72. Its left face has area 48 while its top face has area 96. Find the volume of the box.

3. Solve for x:
$$\log_2 8 - \log_3 9 = \log_5 x$$

4. Find all solutions of
$$20x^2 + 33x - 27 = 0$$

5. Find the sum
$$1 + i + i^2 + i^3 + i^4 + i^5 + \cdots + i^{2016}$$
where $i = \sqrt{-1}$.

6. When the cubic polynomial $x^3 - x^2 + kx - 2$ is divided by $x - 3$ the remainder is k. Find the value of the constant k.

7. Find the maximum value of $x + y$ given that
$$\begin{aligned} 3x + 11y &\leq 198 \\ 5x + y &\leq 70 \end{aligned}$$

8. Find the maximum number of regions of the plane formed by three ellipses lying in that plane.

9. Find the minimum value of the function $f(x, y)$ where
$$f(x, y) = |20 - x| + |x - y| + |y - 50|.$$

10. Let S_n equal the sum of the first n terms of an arithmetic sequence. If $S_{20} = 180$ and $S_{40} = 500$, find the value of S_{60}.

11. A *lattice point* in the plane is a point such that both of its coordinates are integers. How many such lattice points lie on the curve $x^2 + 2y^2 = 81$?

12. Find the sum of all the solutions of the equation

$$\frac{1}{x} - \frac{1}{x+1} = \frac{1}{3x+15}$$

13. Find the sum of the cubes of the roots of

$$x^3 - 11x^2 + 9 = 0$$

14. In how many ways can 2016 be written as the sum of two or more consecutive integers?

15. For some constants a, b, and c, we have that

$$p(x) = x^3 - ax^2 + bx - c$$
$$p(x) = (x-a)(x-b)(x-c)$$

Find the value of $p(4)$.

Spring 2016 McNabb GDCTM Contest
PreCalculus

NO Calculators Allowed

1. Find the prime factorization of $3^8 - 1$.
2. Find a pair of positive integers (m, n) that satisfy $17m - 19n = 1$.
3. Find the maximum value of $11\cos\theta - 2\cos^2\theta$.
4. Ten chairs are set up in a row. In how many ways can three people occupy the chairs so that no two sit next to each other?
5. In how many ways can a class of 12 students be split into three groups of four students each?
6. For all $x \neq 0$, let
$$2f(x) + 5xf(1/x) = 3x + 2$$
Find x if $f(x) = 7$.
7. The longer base of an isosceles trapezoid is equal to a diagonal of the trapezoid. The shorter base of the trapezoid is equal to the altitude of the trapezoid. Find the ratio of the shorter base to the longer base.
8. Find the number of ways to make change for 2 dollars using nickels, dimes, and quarters.
9. Passwords for a certain device must use only the capital letters A, B, or C. The passwords must be exactly of length 8 and each of those three capital letters must be used at least once. How many such passwords are there?
10. Let
$$z + \frac{1}{z} = 2\cos(15^\circ)$$
Find an integer n such that $0 < n < 90$ and
$$z^2 + \frac{1}{z^2} = 2\cos(n^\circ)$$
11. Find a 2×2 matrix M with integer entries that satisfies the equation:
$$M^2 = \begin{pmatrix} 5 & -4 \\ 4 & -3 \end{pmatrix}$$

12. Let the function $f(x,y)$ satisfy the recursive rules

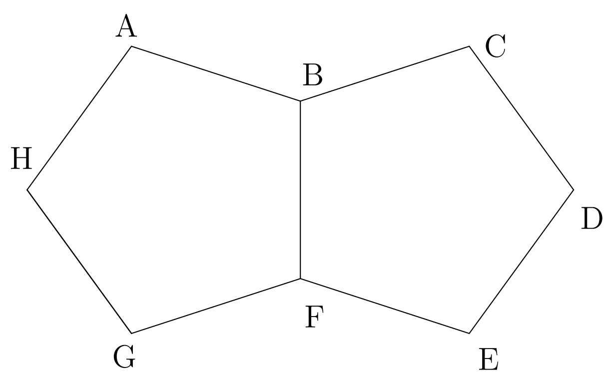
$$f(x, y+1) = f(f(x,y), y) + 4$$
$$f(x, 0) = x$$

Calculate the value of $f(5,5)$

13. Evaluate

$$\frac{\cos 87^\circ}{\sin 1^\circ} - \frac{\sin 87^\circ}{\cos 1^\circ}$$

14. Two regular pentagons, both of side length 2, are glued togther at one edge to form a non-convex octogon $ABCDEFGH$ as shown. What is the value of $(EG)^2$? Your answer must be in the form $a + b\sqrt{c}$ where a, b, and c are positive integers and c has no perfect square factors greater than one.



15. A *lattice point* in the plane is a point such that both of its coordinates are integers. How many such lattice points lie on the curve $x^2 + 2y^2 = 81$?

Spring 2016 McNabb GDCTM Contest
Calculus

NO Calculators Allowed

1. In how many ways can the letters in DALLAS be arranged so that neither the A's nor the L's are next to each other?

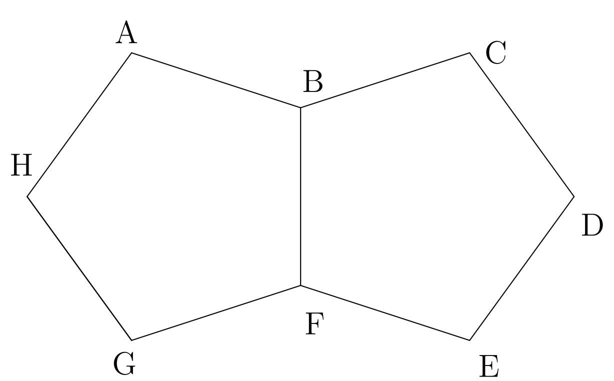
2. Hezy, Zeke, and Elias are running around a track in the same direction. Each of them runs at their own constant pace. Hezy is the fastest and passes Elias every 8 minutes. Meanwhile, Elias passes Zeke every 12 minutes. So how many seconds elapse between times Hezy passes Zeke?

3. If a, b, and c are positive integers satisfying $abc = 1560$, find the least possible value of $a + b + c$.

4. Find the sum
$$1 + i + i^2 + i^3 + i^4 + i^5 + \cdots + i^{2016}$$
where $i = \sqrt{-1}$.

5. Find the maximum number of regions of the plane formed by three ellipses lying in that plane.

6. Two regular pentagons, both of side length 2, are glued togther at one edge to form a non-convex octogon $ABCDEFGH$ as shown. What is the value of $(EG)^2$? Your answer must be in the form $a + b\sqrt{c}$ where a, b, and c are positive integers and c has no perfect square factors greater than one.



7. Find the maximum value of $11 \cos\theta - 2\cos^2\theta$.

8. For what value of n is it true that

$$\int_0^n x^2\,dx = 9$$

?

9. Find the coordinates of a point on the curve $x^2 + xy + y^2 = 3$ at which the curve has a horizontal tangent line.

10. Evaluate

$$\lim_{n\to\infty} \frac{1}{n} \int_0^n \frac{x^4}{3x^4+1}\,dx$$

11. Find the total area enclosed by the polar graph $r^2 = 18\cos(2\theta)$.

12. Evaluate

$$\int_1^{64} \frac{1}{\sqrt{x}(\sqrt{x}+\sqrt[3]{x})}\,dx$$

13. Let

$$f(x) = \frac{2}{x^2+10x+24}$$

Find the value of the sixth derivative of $f(x)$ at the point $x = -5$.

14. Evaluate

$$\sum_{n=0}^{\infty} \frac{(n+1)(n+2)}{2^n}$$

15. Evaluate

$$\int_0^{\infty} \frac{\tan^{-1}(ex) - \tan^{-1}(x)}{x}\,dx$$