

SPRING 2016 McNABB GDCTM CONTEST  
CALCULUS

NO Calculators Allowed

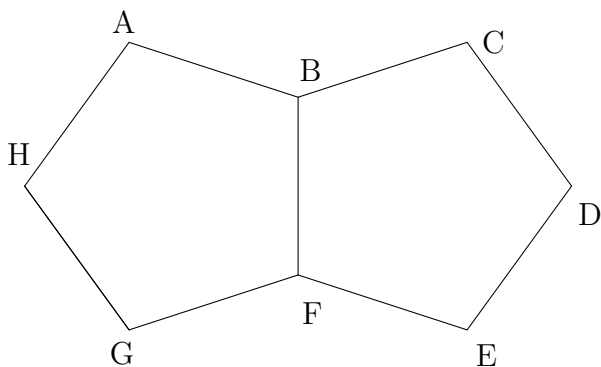
1. In how many ways can the letters in DALLAS be arranged so that neither the A's nor the L's are next to each other?
2. Hezy, Zeke, and Elias are running around a track in the same direction. Each of them runs at their own constant pace. Hezy is the fastest and passes Elias every 8 minutes. Meanwhile, Elias passes Zeke every 12 minutes. So how many seconds elapse between times Hezy passes Zeke?
3. If  $a$ ,  $b$ , and  $c$  are positive integers satisfying  $abc = 1560$ , find the least possible value of  $a + b + c$ .

4. Find the sum

$$1 + i + i^2 + i^3 + i^4 + i^5 + \dots + i^{2016}$$

where  $i = \sqrt{-1}$ .

5. Find the maximum number of regions of the plane formed by three ellipses lying in that plane.
6. Two regular pentagons, both of side length 2, are glued together at one edge to form a non-convex octagon  $ABCDEFGH$  as shown. What is the value of  $(EG)^2$ ? Your answer must be in the form  $a + b\sqrt{c}$  where  $a$ ,  $b$ , and  $c$  are positive integers and  $c$  has no perfect square factors greater than one.



7. Find the maximum value of  $11 \cos \theta - 2 \cos^2 \theta$ .

8. For what value of  $n$  is it true that

$$\int_0^n x^2 dx = 9$$

?

9. Find the coordinates of a point on the curve  $x^2 + xy + y^2 = 3$  at which the curve has a horizontal tangent line.

10. Evaluate

$$\lim_{n \rightarrow \infty} \frac{1}{n} \int_0^n \frac{x^4}{3x^4 + 1} dx$$

11. Find the total area enclosed by the polar graph  $r^2 = 18 \cos(2\theta)$ .

12. Evaluate

$$\int_1^{64} \frac{1}{\sqrt{x}(\sqrt{x} + \sqrt[3]{x})} dx$$

13. Let

$$f(x) = \frac{2}{x^2 + 10x + 24}$$

Find the value of the sixth derivative of  $f(x)$  at the point  $x = -5$ .

14. Evaluate

$$\sum_{n=0}^{\infty} \frac{(n+1)(n+2)}{2^n}$$

15. Evaluate

$$\int_0^{\infty} \frac{\tan^{-1}(ex) - \tan^{-1}(x)}{x} dx$$