Spring 2016 McNabb GDCTM Contest
PreCalculus

NO Calculators Allowed

1. Find the prime factorization of $3^8 - 1$.

2. Find a pair of positive integers (m, n) that satisfy $17m - 19n = 1$.

3. Find the maximum value of $11\cos\theta - 2\cos^2\theta$.

4. Ten chairs are set up in a row. In how many ways can three people occupy the chairs so that no two sit next to each other?

5. In how many ways can a class of 12 students be split into three groups of four students each?

6. For all $x \neq 0$, let
$$2f(x) + 5xf(1/x) = 3x + 2$$
Find x if $f(x) = 7$.

7. The longer base of an isosceles trapezoid is equal to a diagonal of the trapezoid. The shorter base of the trapezoid is equal to the altitude of the trapezoid. Find the ratio of the shorter base to the longer base.

8. Find the number of ways to make change for 2 dollars using nickels, dimes, and quarters.

9. Passwords for a certain device must use only the capital letters A, B, or C. The passwords must be exactly of length 8 and each of those three capital letters must be used at least once. How many such passwords are there?

10. Let
$$z + \frac{1}{z} = 2\cos(15^\circ)$$
Find an integer n such that $0 < n < 90$ and
$$z^2 + \frac{1}{z^2} = 2\cos(n^\circ)$$

11. Find a 2×2 matrix M with integer entries that satisfies the equation:
$$M^2 = \begin{pmatrix} 5 & -4 \\ 4 & -3 \end{pmatrix}$$

12. Let the function $f(x, y)$ satisfy the recursive rules

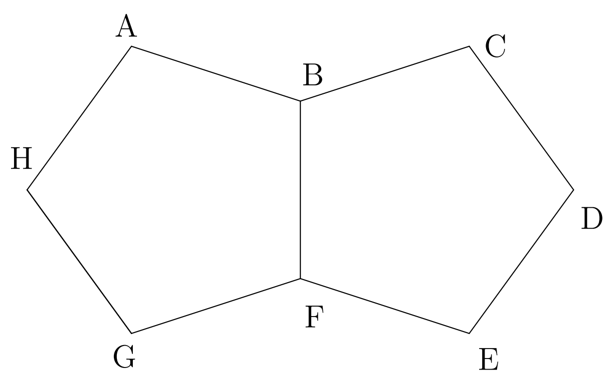
$$f(x, y+1) = f(f(x, y), y) + 4$$
$$f(x, 0) = x$$

Calculate the value of $f(5, 5)$

13. Evaluate

$$\frac{\cos 87^\circ}{\sin 1^\circ} - \frac{\sin 87^\circ}{\cos 1^\circ}$$

14. Two regular pentagons, both of side length 2, are glued togther at one edge to form a non-convex octogon $ABCDEFGH$ as shown. What is the value of $(EG)^2$? Your answer must be in the form $a + b\sqrt{c}$ where a, b, and c are positive integers and c has no perfect square factors greater than one.



15. A *lattice point* in the plane is a point such that both of its coordinates are integers. How many such lattice points lie on the curve $x^2 + 2y^2 = 81$?