

SPRING 2016 MCNABB GDCTM CONTEST
ALGEBRA II

NO Calculators Allowed

1. A thirteen foot tree is growing at a rate of three feet per year while a forty-one foot tree is growing at a rate of two feet per year. In how many years will the two trees be the same height?

2. The front face of a rectangular box has area 72. Its left face has area 48 while its top face has area 96. Find the volume of the box.

3. Solve for x :

$$\log_2 8 - \log_3 9 = \log_5 x$$

4. Find all solutions of

$$20x^2 + 33x - 27 = 0$$

5. Find the sum

$$1 + i + i^2 + i^3 + i^4 + i^5 + \dots + i^{2016}$$

where $i = \sqrt{-1}$.

6. When the cubic polynomial $x^3 - x^2 + kx - 2$ is divided by $x - 3$ the remainder is k . Find the value of the constant k .

7. Find the maximum value of $x + y$ given that

$$3x + 11y \leq 198$$

$$5x + y \leq 70$$

8. Find the maximum number of regions of the plane formed by three ellipses lying in that plane.

9. Find the minimum value of the function $f(x, y)$ where

$$f(x, y) = |20 - x| + |x - y| + |y - 50|.$$

10. Let S_n equal the sum of the first n terms of an arithmetic sequence. If $S_{20} = 180$ and $S_{40} = 500$, find the value of S_{60} .

11. A *lattice point* in the plane is a point such that both of its coordinates are integers. How many such lattice points lie on the curve $x^2 + 2y^2 = 81$?

12. Find the sum of all the solutions of the equation

$$\frac{1}{x} - \frac{1}{x+1} = \frac{1}{3x+15}$$

13. Find the sum of the cubes of the roots of

$$x^3 - 11x^2 + 9 = 0$$

14. In how many ways can 2016 be written as the sum of two or more consecutive integers?

15. For some constants a , b , and c , we have that

$$\begin{aligned} p(x) &= x^3 - ax^2 + bx - c \\ p(x) &= (x - a)(x - b)(x - c) \end{aligned}$$

Find the value of $p(4)$.