Fall 2012 McNabb GDCTM Contest
Algebra Two

NO Calculators Allowed

1. How many numbers are in the list

$$21, 13, 5, -3, -11, \cdots, -203, -211$$

   where each number is 8 less than the one before it?

   (A) 20   **(B)** 23   **(C)** 28   **(D)** 29   **(E)** 30

2. What is the largest possible value of the greatest common factor of six different two-digit whole numbers?

   (A) 10   **(B)** 12   **(C)** 15   **(D)** 16   **(E)** 19

3. In the sequence of numbers

$$a, b, 1, -1, 0, -1, -1, -2, \cdots$$

   each number after the second is the sum of the previous two numbers. Find the value of a.

   (A) -1   **(B)** 3   **(C)** 0   **(D)** 4   **(E)** 1

4. A certain triangle in the coordinate plane has area 6. Then the x coordinates of each vertex of this triangle are doubled, but the y coordinates of each vertex are left alone. What is the area of this new triangle?

   (A) 3   **(B)** 6   **(C)** 12   **(D)** 24   **(E)** cannot be determined

5. The points x, x^2, and x^3 are graphed on the number line below. Which could be the value of x?

   (A) -2   **(B)** -1   **(C)** -1/2   **(D)** 1/3   **(E)** 2

6. There are two non-congruent triangles ABC with $AB = 8$, $BC = 5$, and $\angle A = 30°$. What is the positive difference of their areas?

   (A) 5   **(B)** 6   **(C)** $3\sqrt{3}$ **(D)** $5\sqrt{3}$ **(E)** 12

7. In how many ways can the letters in CHEETAH be arranged so that no two consecutive letters are the same?

   (A) 660   **(B)** 540   **(C)** 1260   **(D)** 720   **(E)** 330

8. The coefficient of x^{18} in the product

$$(x+1)(x+3)(x+5)(x+7)\cdots(x+37)$$

   is equal to

   (A) 1   **(B)** 243   **(C)** 361   **(D)** 400   **(E)** 401

9. On the first test of the school year an algebra class averaged 81. If the three lowest scoring exams were not considered, the average would have been 84. If those three lowest scores were 52, 62, and 66, how many students are in the algebra class?

   (A) 21   **(B)** 24   **(C)** 26   **(D)** 27   **(E)** 28

10. In $\triangle ABC$, points D and E lie on sides AC and AB respectively. Draw BD and CE intersecting at point F. Suppose $AC = AB = 12$, $BF = FC = 6$, and $EF = FD = 5$. Find the length of AD.

    (A) 7   **(B)** 9   **(C)** 10   **(D)** 11   **(E)** 12

11. If the point $(1, -5)$ lies on the graph of $y = -f(1-2x)+2$ which point below must lie on the graph of $y = 3f(5x-6)-8$?

    (A) $(1, 13)$ **(B)** $(3, -8)$ **(C)** $(-1, -8)$ **(D)** $(0, 11)$ **(E)** $(-4, 8)$

12. How many real solutions are there to the equation

$$(x+1)(x+2)(x+3)(x+4) = (x+5)(x+6)(x+7)(x+8)$$

    ?

    (A) 0   **(B)** 1   **(C)** 2   **(D)** 3   **(E)** 4

13. How many ordered pairs (x, y) of positive integers satisfy both

$$\frac{x}{8} + \frac{y}{3} > 1 \quad \text{and} \quad \frac{x}{12} + \frac{y}{7} < 1$$

   (A) 22   (B) 23   (C) 24   (D) 25   (E) 26

14. A boat goes downriver from A to B in 3 days and returns upriver from B to A in 4 days. How long in days would it take an inner tube to float downriver from A to B?

   (A) 12   (B) 18   (C) 24   (D) 30   (E) 32

15. Which of the following are true for all positive real numbers a and b?
   I. $\sqrt{ab} = \sqrt{a}\sqrt{b}$
   II. $\sqrt{a} + \sqrt{b} > \sqrt{a+b}$
   III. $\sqrt{\dfrac{a^2 + b^2}{2}} < \dfrac{a+b}{2}$
   (A) I only   (B) I and II only   (C) I and III only
   (D) II and III only   (E) I, II, and III

16. Three congruent circles are mutually externally tangent to each other and each circle is tangent to a different side of an equilateral triangle of side length 2. The common radius of the circles can be written in the form $a - \sqrt{b}$, where a and b are positive integers. What is the value of $3a + b$?

   (A) 7   (B) 8   (C) 9   (D) 10   (E) 11

17. Find a number c so that the three distinct solutions $x_1 < x_2 < x_3$ of the equation $x^3 + 6x^2 - 8x + 4 = c$ satisfy $x_1 + x_3 = 2x_2$.

   (A) 36   (B) 37   (C) 38   (D) 39   (E) 40

18. Find the value of x if

$$3x + 2y - z = 1$$
$$-x + y - 3z = 7$$
$$x + 2y + 9z = -1$$

   (A) -2   (B) -1   (C) 0   (D) 1   (E) 2

19. At how many points do the graphs of $y = x^4 + x^3 - 2x^2 + x$ and $y = x$ intersect?

   (A) 1   **(B)** 2   **(C)** 3   **(D)** 4   **(E)** 5

20. A caravan of 5 cars is headed down I-35 for the annual Texas A&M High School Math Tournament. In how many ways can the caravan be re-formed after a rest stop so that each non-leading car has a different car in front of it from the one it had before the rest stop? Note the car that lead before the rest stop may still be leading after it and also could be any of the following cars after it as well.

   (A) 31   **(B)** 53   **(C)** 60   **(D)** 65   **(E)** 71