Fall 2012 McNabb GDCTM Contest
Geometry

NO Calculators Allowed

1. If one defines
$$(a,b) \wedge (c,d) = ad - bc$$
solve this equation for x: $(2,x) \wedge (7,-4) = 3$

(A) $-\frac{7}{11}$ **(B)** $\frac{11}{7}$ **(C)** $\frac{7}{11}$ **(D)** 11 **(E)** $-\frac{11}{7}$

2. A certain triangle in the coordinate plane has area 6. Then the x coordinates of each vertex of this triangle are doubled, but the y coordinates of each vertex are left alone. What is the area of this new triangle?

(A) 3 **(B)** 6 **(C)** 12 **(D)** 24 **(E)** cannot be determined

3. The points x, x^2, and x^3 are graphed on the number line below. Which could be the value of x?

(A) -2 **(B)** -1 **(C)** -1/2 **(D)** 1/3 **(E)** 2

4. In how many ways can the letters in CHEETAH be arranged so that no two consecutive letters are the same?

(A) 660 **(B)** 540 **(C)** 1260 **(D)** 720 **(E)** 330

5. What is the area of a rhombus with sides equal to 13 and short diagonal equal to 10?

(A) 60 **(B)** 65 **(C)** 120 **(D)** 130 **(E)** 260

6. In how many ways can 9 students be divided into 3 groups of 3 students each?

(A) 81 **(B)** 180 **(C)** 280 **(D)** 540 **(E)** 1680

7. In $\triangle ABC$, points D and E lie on sides AC and AB respectively. Draw BD and CE intersecting at point F. Suppose $AC = AB = 12$, $BF = FC = 6$, and $EF = FD = 5$. Find the length of AD.

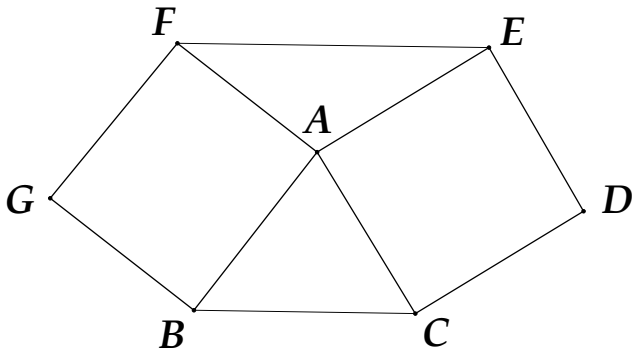
   (A) 7 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

8. On the first test of the school year an algebra class averaged 81. If the three lowest scoring exams were not considered, the average would have been 84. If those three lowest scores were 52, 62, and 66, how many students are in the algebra class?

   (A) 21 **(B)** 24 **(C)** 26 **(D)** 27 **(E)** 28

9. An equilateral triangle ABC fits between two squares, $AEDC$ and $ABGF$ as shown. Segment FE is drawn. What is the measure of $\angle AFE$ in degrees?

   (A) 30 **(B)** 45 **(C)** 50 **(D)** 55 **(E)** 60



10. A problem from the *Liber Abaci*, a math text written by Fibonnaci in the 13th century:

    > On a certain ground there are two towers, one of which is 30 feet high, the other 40, and they are only 50 feet apart; two birds descending together from the heights of the two towers fly to the center of a fountain between the towers; the distance from the center [of the fountain] to the foot of the higher tower is sought.

    In this problem assume: the birds are flying at the same speed, depart at the same time, and arrive together at the fountain; and the fountain and feet of the towers are collinear.

    (A) 18 **(B)** 20 **(C)** 22 **(D)** 24 **(E)** 32

11. Sixty points are equally spaced entirely around a circle. How many regular polygons can be formed using these and only these points as vertices?

(A) 60 **(B)** 68 **(C)** 78 **(D)** 88 **(E)** 89

12. There are two non-congruent triangles ABC with $AB = 8$, $BC = 5$, and $\angle A = 30^\circ$. What is the positive difference of their areas?

(A) 5 **(B)** 6 **(C)** $3\sqrt{3}$ **(D)** $5\sqrt{3}$ **(E)** 12

13. The coefficient of x^{18} in the product

$$(x+1)(x+3)(x+5)(x+7)\cdots(x+37)$$

is equal to

(A) 1 **(B)** 243 **(C)** 361 **(D)** 400 **(E)** 401

14. Let $a \neq b$. The equation of the perpendicular bisector of the segment with endpoints (a, b) and (b, a) is

(A) $y = x$ **(B)** $y = 0$ **(C)** $x = 0$ **(D)** $y = -x$ **(E)** $y = 2x$

15. Suppose that the statements:

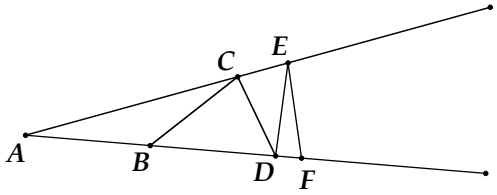
No *zoofs* are *zarns*
At least one *zune* is not a *zoof*

are true. Which of the following must be true?

(A) At least one *zune* is a *zoof*
(B) No *zarn* is a *zune*
(C) At least one *zarn* is not a *zune*
(D) All *zunes* are *zarns*
(E) None of the above

16. Points B, C, D, E, and F lie as shown on $\angle A$ with $AB = BC = CD = DE = EF$ as shown. If $\angle AEF$ is right, then find, in degrees, the measure of $\angle CAB$.

(A) 16 **(B)** 18 **(C)** 20 **(D)** 22 **(E)** 24



17. The line $4x - 8y = 15$ is irritating to graph as it contains no points of the form (a, b), with both a and b integers. Such points are called *lattice points*. What is the minimum distance between this line and the set of lattice points?

(A) $\frac{1}{12}$ **(B)** $\frac{1}{10}$ **(C)** $\frac{\sqrt{5}}{20}$
(D) $\frac{\sqrt{5}}{15}$ **(E)** no minimum exists

18. The real number $\sqrt{41 - 24\sqrt{2}}$ can be put in the form $a\sqrt{2} - b$ where a and b are positive integers. What is the value of $a + b$?

(A) 4 **(B)** 7 **(C)** 8 **(D)** 16 **(E)** 18

19. An ordered triple of positive integers (a, b, c) with $a < b < c$ and $a^2 + b^2 = c^2$ is called *Pythagorean*. Find the perimeter of the only Pythagorean triple with $a = 11$.

(A) 88 **(B)** 99 **(C)** 121 **(D)** 132 **(E)** 154

20. The trapezoid $ABCD$ has $AB \parallel CD$, $AB = 5$, and $DC = 12$. Draw EF parallel to AB with E on AD and F on BC. If EF splits trapezoid $ABCD$ into two trapezoids of equal area, what is the length of EF?

(A) 9 **(B)** $\frac{120}{17}$ **(C)** $\frac{17}{2}$ **(D)** $\frac{13\sqrt{2}}{2}$ **(E)** $2\sqrt{15}$