Spring 2011 McNabb GDCTM Contest
Pre-Algebra

NO Calculators Allowed

1. Hezy leaves home for work at 6:45am. He drives to the Green Line train station 3 miles away at an average speed of 30 mph. After 8 minutes he boards the train for downtown. The train averages 45 mph for its 9 mile journey. After a 7 minute walk Hezy arrives at work. What time does Hezy arrive at work?

   (A) 7:11am **(B)** 7:18am **(C)** 7:21am **(D)** 7:27am **(E)** 7:29am

2. If today is a Saturday, what day of the week will it be 1001 days from today?

   (A) Thursday **(B)** Friday **(C)** Saturday
   (D) Sunday **(E)** Monday

3. The sum of all the factors of 1001 is equal to

   (A) 1002 **(B)** 1344 **(C)** 1440 **(D)** 1836 **(E)** 2002

4. How many arrangements of $REVERE$ are there in which the first R occurs before the first E?

   (A) 12 **(B)** 18 **(C)** 20 **(D)** 24 **(E)** 30

5. In a class, $2/3$ of the students have brown eyes and $4/5$ of the students have brown hair. If students with brown eyes are twice as likely to have brown hair as students who do not have brown eyes, what fraction of the class has neither brown eyes nor brown hair?

   (A) $1/30$ **(B)** $1/15$ **(C)** $1/10$ **(D)** $2/15$ **(E)** $1/5$

6. When three different numbers from the set $\{-7, -2, -1, 0, 4, 5\}$ are multiplied together the smallest possible product is

   (A) -343 **(B)** -175 **(C)** -140 **(D)** -14 **(E)** 0

7. Out of a sphere of clay with diameter 12, Marty fashions two spheres of radius 3 and 5 respectively. Using all of the remaining clay Jennifer fashions a sphere. What is the diameter of Jennifer's sphere?

(A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

8. Let m and n be integers satisfying $m^2 + n^2 = 50$. The value of $m + n$ must be

(A) -8 **(B)** -5 **(C)** 0
(D) 10 **(E)** cannot be uniquely determined

9. Suppose that \$600 is divided into two parts in the ratio of 2 : 3. The larger of these parts is then further subdivided into two parts in the ratio of 3 : 2. The smallest of these now three parts is

(A) \$96 **(B)** \$144 **(C)** \$192 **(D)** \$216 **(E)** \$240

10. Points A, B, and C all lie on the same straight line and occur in the order given. If $AB/BC = 2/5$ and $AC = 35$, what is AB?

(A) 7 **(B)** 10 **(C)** 15 **(D)** 21 **(E)** 25

11. Suppose the integer n has prime factorization $2^6 \cdot 3^8$. How many perfect square factors does n have?

(A) 12 **(B)** 18 **(C)** 20 **(D)** 24 **(E)** 48

12. The product $60 \times 60 \times 24 \times 7$ equals

(A) the number of minutes in seven weeks
(B) the number of hours in sixty days
(C) the number of seconds in seven hours
(D) the number of seconds in one week
(E) the number of minutes in twenty-four weeks

13. The four digit integer $1A8B$ is divisible by 77. What is the value of $A + B$?

(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

14. The surface area of sphere A is 50. Its volume is $\frac{1}{27}$th of the volume of sphere B. What is the surface area of sphere B?

(A) 50/9 **(B)** 50/3 **(C)** 50 **(D)** 150 **(E)** 450

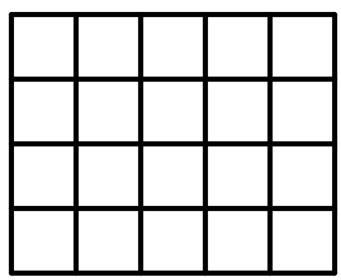
15. If n and m are positive integers and $480n = m^2$, what is the smallest possible value of m?

(A) 90 **(B)** 120 **(C)** 180 **(D)** 240 **(E)** 480

16. A fifth number n is added to the set $\{3, 6, 9, 10\}$ to form a new set $\{3, 6, 9, 10, n\}$. For how many values of n is the mean of this new set equal to its own median?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** more than 3

17. How many rectangles are in this figure?



(A) 150 **(B)** 300 **(C)** 600 **(D)** 640 **(E)** 800

18. A regular 52 card deck is well shuffled. What is the probability that both the top and bottom cards are aces?

(A) 1/26 **(B)** 1/52 **(C)** 3/221 **(D)** 2/221 **(E)** 1/221

19. In two years a son will be one-third as old as his father was 2 years ago. In eighteen years this son will be the same age as his father was 18 years ago. How old is the son now?

(A) 10 **(B)** 12 **(C)** 14 **(D)** 16 **(E)** 18

20. How many different rectangular prisms can be made using exactly 48 unit cubes? Two prisms are the same if one can be rotated to coincide with the other. For example, a $3 \times 4 \times 4$ rectangular prism is the same as a $4 \times 3 \times 4$ rectangular prism.

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

21. Recall that a Pythagorean triple is a triple (a, b, c) of positive integers satisfying $a^2 + b^2 = c^2$. Which of the following must be true?

(I.) At least one of a, b, and c must be odd

(II.) At least one of a, b, and c must be even

(III.) For at least one Pythagorean triple, $a = b$.

(A) I only **(B)** II only **(C)** I and II only
(D) II and III only **(E)** none of them

22. The twelve edges of a cube are marked with the integers 1 through 12 in such a way that each edge receives a different number. Then each vertex of the cube is assigned the number equal to the sum of the numbers on the edges that meet at that vertex. Finally each face of the cube is assigned the sum of the numbers at each vertex of that face. What must be the sum of all the numbers that have been assigned to the faces of the cube?

(A) 156 **(B)** 234 **(C)** 390
(D) 468 **(E)** cannot be uniquely determined

23. Councilman Bob sits on a 12 member City Council. A committee of 4 councilpeople is to be selected at random. What is probability that Bob is selected to be on this committee?

(A) 1/12 **(B)** 1/4 **(C)** 1/3 **(D)** 4/9 **(E)** 1/2

24. In a class of 28 students, 20 take Latin, 14 take Greek, and 10 take Hebrew. If no student takes all three languages and 6 take no language, how many students must be taking both Greek and Hebrew?

(A) 0 **(B)** 1 **(C)** 2
(D) 3 **(E)** cannot be uniquely determined

25. Molly's Motel is adopting a new room key system. The new keys will be square 3×3 cards each with two holes punched in them as in the figure. The two sides (what we would have called the front and back except we cannot tell which is which!) of such a card cannot be distinguished but there is a distinguished edge which is the edge to be inserted in the lock. What is the greatest number of rooms Molly's Motel can have?

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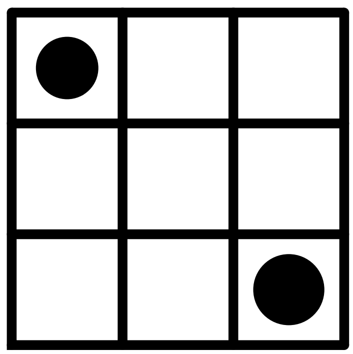


(A) 18 **(B)** 21 **(C)** 24 **(D)** 30 **(E)** 36

Spring 2011 McNabb GDCTM Contest
Algebra I

NO Calculators Allowed

1. Hezy leaves home for work at 6:45am. He drives to the Green Line train station 3 miles away at an average speed of 30 mph. After 8 minutes he boards the train for downtown. The train averages 45 mph for its 9 mile journey. After a 7 minute walk Hezy arrives at work. What time does Hezy arrive at work?

   (A) 7:11am **(B)** 7:18am **(C)** 7:21am **(D)** 7:27am **(E)** 7:29am

2. How many arrangements of $REVERE$ are there in which the first R occurs before the first E?

   (A) 12 **(B)** 18 **(C)** 20 **(D)** 24 **(E)** 30

3. If $a \blacktriangle b = b(a+1)$ what is the value of $(a \blacktriangle 1) \blacktriangle (1 \blacktriangle a)$?

   (A) $2a^2 + 4a$ **(B)** $2a^2 + 3a + 1$ **(C)** $a^2 + 3a + 2$
   (D) $a^2 + 3a$ **(E)** $6a$

4. Suppose that \$600 is divided into two parts in the ratio of 2 : 3. The larger of these parts is then further subdivided into two parts in the ratio of 3 : 2. The smallest of these now three parts is

   (A) \$96 **(B)** \$144 **(C)** \$192 **(D)** \$216 **(E)** \$240

5. A fifth number n is added to the set $\{3, 6, 9, 10\}$ to form a new set $\{3, 6, 9, 10, n\}$. For how many values of n is the mean of this new set equal to its own median?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** more than 4

6. How many different rectangular prisms can be made using exactly 48 unit cubes?

   (A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

7. In a class, 2/3 of the students have brown eyes and 4/5 of the students have brown hair. If students with brown eyes are twice as likely to have brown hair as students who do not have brown eyes, what fraction of the class has neither brown eyes nor brown hair?

(A) 1/30 **(B)** 1/15 **(C)** 1/10 **(D)** 2/15 **(E)** 1/5

8. When three different numbers from the set $\{-7, -2, -1, 0, 4, 5\}$ are multiplied together the smallest possible product is

(A) -343 **(B)** -175 **(C)** -140 **(D)** -14 **(E)** 0

9. Out of a sphere of clay with diameter 12, Marty fashions two spheres of radius 3 and 5 respectively. Using all of the remaining clay Jennifer fashions a sphere. What is the diameter of Jennifer's sphere?

(A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

10. The product $60 \times 60 \times 24 \times 7$ equals

(A) the number of minutes in seven weeks
(B) the number of hours in sixty days
(C) the number of seconds in seven hours
(D) the number of seconds in one week
(E) the number of minutes in twenty-four weeks

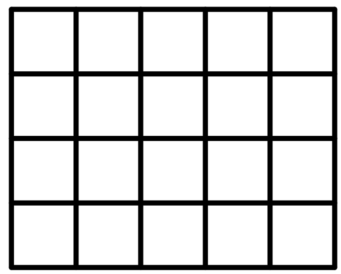
11. Let a, b, x, and $y > 0$. If $x = by$ and $y = ax$ find the value of $\dfrac{a}{1+a} + \dfrac{b}{1+b}$.

(A) 1 **(B)** a **(C)** b/a **(D)** 2 **(E)** $1/(a+b)$

12. If n and m are positive integers and $480n = m^2$, what is the smallest possible value of m?

(A) 90 **(B)** 96 **(C)** 120 **(D)** 240 **(E)** 480

13. How many rectangles are in this figure?



(A) 20 **(B)** 75 **(C)** 150 **(D)** 300 **(E)** 600

14. The sum of two positive numbers is S and their positive difference is $1/m$th of the smaller number. What is the value of the larger number?

(A) $\frac{mS}{2m+1}$ **(B)** $\frac{(m-1)S}{2m}$ **(C)** $\frac{m^2S}{2m-1}$ **(D)** $\frac{2mS}{m+1}$ **(E)** $\frac{(m+1)S}{2m+1}$

15. A regular 52 card deck is well shuffled. What is the probability that both the top and bottom cards are aces?

(A) $1/26$ **(B)** $1/52$ **(C)** $3/221$ **(D)** $2/221$ **(E)** $1/221$

16. In two years a son will be one-third as old as his father was 2 years ago. In eighteen years this son will be the same age as his father was 18 years ago. How old is the son now?

(A) 10 **(B)** 12 **(C)** 14 **(D)** 16 **(E)** 18

17. Let $f(x)$ be a linear function satisfying $f(0) = 0$. If both $f(a+b) = 7$ and $f(a-2b) = 3$, then the value of $f(a+7b)$ must be

(A) 9 **(B)** 11 **(C)** 13
(D) 15 **(E)** cannot be uniquely determined

18. Recall that a Pythagorean triple is a triple (a, b, c) of positive integers satisfying $a^2 + b^2 = c^2$. Which of the following must be true?

(I.) At least one of a, b, and c must be odd

(II.) At least one of a, b, and c must be even

(III.) For at least one Pythagorean triple, $a = b$.

(A) I only **(B)** II only **(C)** I and II only
(D) II and III only **(E)** none of them

19. A train having to journey x miles in h hours, ran for k hours at a rate of r miles per hour, then stopped for m minutes. How fast must it go (in mph) on the rest of its journey to arrive on time?

(A) $\dfrac{x - kr}{h - k - m}$ **(B)** $\dfrac{x - kr}{60h - 60k - m}$ **(C)** $\dfrac{60(x - kr)}{h - k - m}$
(D) $\dfrac{60(x - kr)}{h - k - 60m}$ **(E)** $\dfrac{60(x - kr)}{60h - 60k - m}$

20. The image of the line $y = 4x - 6$ under reflection across the line $y = -x$ is the line

(A) $y = \frac{1}{4}x - \frac{3}{2}$ **(B)** $y = -\frac{1}{4}x + \frac{3}{2}$ **(C)** $y = \frac{1}{4}x - \frac{4}{3}$ **(D)** $y = \frac{1}{4}x - 1$
(E) $y = \frac{1}{4}x + \frac{2}{3}$

21. Let m and n be integers satisfying $m^2 + n^2 = 50$. The value of $m + n$ must be

(A) -8 **(B)** -5 **(C)** 0
(D) 10 **(E)** cannot be uniquely determined

22. In a class of 28 students, 20 take Latin, 14 take Greek, and 10 take Hebrew. If no student takes all three languages and 6 take no language, how many students must be taking both Greek and Hebrew?

(A) cannot be uniquely determined **(B)** 0 **(C)** 1
(D) 2 **(E)** 3

23. The area of rectangle $ABCD$ is 40. Point P is on AB so that $BP = 3$. Point R is on AD so that $DR = 2$. Given that $APQR$ is a rectangle with area of 15, find the average of the two possible values for the length of AP.

(A) 19/4 **(B)** 21/4 **(C)** 19/2 **(D)** 21/2 **(E)** 5

24. Molly's Motel is adopting a new room key system. The new keys will be square 3×3 cards each with two holes punched in them as in the figure. The two sides (what we would have called the front and back except we cannot tell which is which!) of such a card cannot be distinguished but there is a distinguished edge which is the edge to be inserted in the lock. What is the greatest number of rooms Molly's Motel can have?

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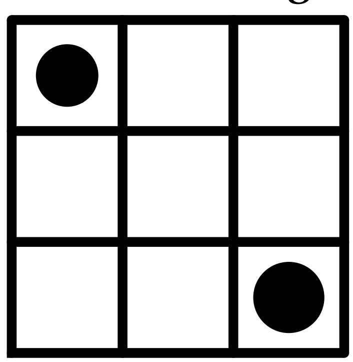

(A) 12 **(B)** 18 **(C)** 21 **(D)** 24 **(E)** 36

25. What is the remainder when $x^{14} + x^{11} + x^8 + x^5 + x^3 + x^2 + x + 1$ is divided by $x^2 - x + 1$?

(A) 3 **(B)** $2x$ **(C)** $4x + 1$ **(D)** $2x - 1$ **(E)** $-x + 4$

Spring 2011 McNabb GDCTM Contest
Geometry

NO Calculators Allowed

1. Hezy leaves home for work at 6:45am. He drives to the Green Line train station 3 miles away at an average speed of 30 mph. After 8 minutes he boards the train for downtown. The train averages 45 mph for its 9 mile journey. After a 7 minute walk Hezy arrives at work. What time does Hezy arrive at work?

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   (A) 1/30 **(B)** 1/15 **(C)** 1/10 **(D)** 2/15 **(E)** 1/5

4. Let m and n be integers satisfying $m^2 + n^2 = 50$. The value of $m + n$ must be

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5. Out of a sphere of clay with diameter 12, Marty fashions two spheres of radius 3 and 5 respectively. Using all of the remaining clay Jennifer fashions a sphere. What is the diameter of Jennifer's sphere?

   (A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

6. Let a, b, x, and $y > 0$. If $x = by$ and $y = ax$ find the value of $\dfrac{a}{1+a} + \dfrac{b}{1+b}$.

   (A) 1 **(B)** a **(C)** b/a **(D)** 2 **(E)** $1/(a+b)$

7. If n and m are positive integers and $480n = m^2$, what is the smallest possible value of m?

(A) 90 **(B)** 96 **(C)** 120 **(D)** 240 **(E)** 480

8. In an isosceles trapezoid with bases 6 and 16 a circle is inscribed (touching all four sides of the trapezoid). What is the diameter of this circle?

(A) 9 **(B)** $4\sqrt{5}$ **(C)** $4\sqrt{6}$ **(D)** 10 **(E)** 11

9. In the coordinate plane a triangle has vertices at $(0, 0)$, $(28, 0)$, and $(8, 15)$. The circle inscribed in this triangle has center at (a, b). What is the value of $a + b$?

(A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

10. A regular 52 card deck is well shuffled. What is the probability that both the top and bottom cards are aces?

(A) $1/26$ **(B)** $1/52$ **(C)** $3/221$ **(D)** $2/221$ **(E)** $1/221$

11. In two years a son will be one-third as old as his father was 2 years ago. In eighteen years this son will be the same age as his father was 18 years ago. How old is the son now?

(A) 10 **(B)** 12 **(C)** 14 **(D)** 16 **(E)** 18

12. In $\triangle ABC$, point D is on segment $\overline{AC}$ and point E is on segment $\overline{AB}$. Segments $\overline{BD}$ and $\overline{CE}$ are drawn, intersecting at piont F. If $AE/EB = 2/5$ and $EF/FC = 3/7$ then the ratio BF/FD is equal to

(A) $7/2$ **(B)** $8/3$ **(C)** 4
(D) $9/2$ **(E)** cannot be determined uniquely

13. Triangle ABC is inscribed in a circle and $AB = AC = 6$. Point D lies on BC with $AD = 4$. AD is extended through D to E on the circle. Find DE.

(A) 4 **(B)** 5 **(C)** 6
(D) 7 **(E)** cannot be uniquely determined

14. Recall that a Pythagorean triple is a triple (a, b, c) of positive integers satisfying $a^2 + b^2 = c^2$. Which of the following must be true?

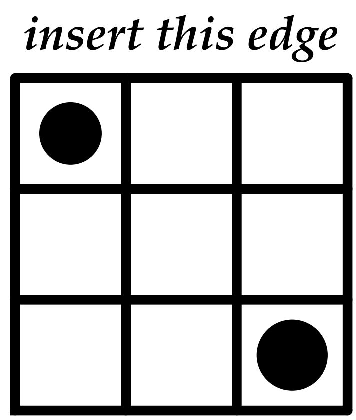
(I.) At least one of a, b, and c must be odd

(II.) At least one of a, b, and c must be even

(III.) For at least one Pythagorean triple, $a = b$.

(A) I only **(B)** II only **(C)** I and II only
(D) II and III only **(E)** none of them

15. Molly's Motel is adopting a new room key system. The new keys will be square 3×3 cards each with two holes punched in them as in the figure. The two sides (what we would have called the front and back except we cannot tell which is which!) of such a card cannot be distinguished but there is a distinguished edge which is the edge to be inserted in the lock. What is the greatest number of rooms Molly's Motel can have?



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16. A train having to journey x miles in h hours, ran for k hours at a rate of r miles per hour, then stopped for m minutes. How fast must it go (in mph) on the rest of its journey to arrive on time?

(A) $\dfrac{x - kr}{h - k - m}$ **(B)** $\dfrac{x - kr}{60h - 60k - m}$ **(C)** $\dfrac{60(x - kr)}{h - k - m}$
(D) $\dfrac{60(x - kr)}{h - k - 60m}$ **(E)** $\dfrac{60(x - kr)}{60h - 60k - m}$

17. The series

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + 100 \cdot 101$$

has the value

(A) 333300 **(B)** 343400 **(C)** 353500 **(D)** 363600 **(E)** 404000

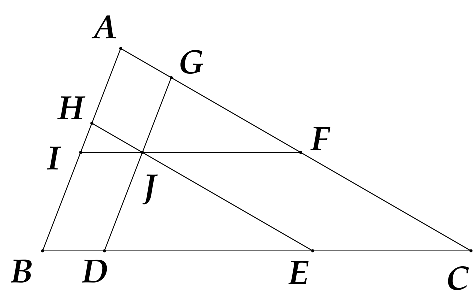
18. In a class of 28 students, 20 take Latin, 14 take Greek, and 10 take Hebrew. If no student takes all three languages and 6 take no language, how many students must be taking both Greek and Hebrew?

(A) cannot be uniquely determined **(B)** 0 **(C)** 1
(D) 2 **(E)** 5

19. Ten coplanar lines are given such that (i) exactly three lines are parallel (each to the other two), (ii) no other pairs of lines are parallel, and (iii) no three lines are concurrent. How many triangles are formed by these ten lines?

(A) 64 **(B)** 81 **(C)** 87 **(D)** 98 **(E)** 100

20. In triangle ABC the transversals DG, EH, and FI are concurrent at J, with $DG \parallel AB$, $EH \parallel AC$, and $FI \parallel BC$. If these three transversals have the same length, what is their common length if it is known that $AB = 8$, $BC = 16$, and $CA = 12$?



(A) 91/13 **(B)** 92/13 **(C)** 94/14 **(D)** 95/14 **(E)** 96/13

Spring 2011 McNabb GDCTM Contest
Algebra II

NO Calculators Allowed

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   (A) 1 **(B)** a **(C)** b/a **(D)** 2 **(E)** $1/(a+b)$

5. If n and m are positive integers and $480n = m^2$, what is the smallest possible value of m?

   (A) 90 **(B)** 96 **(C)** 120 **(D)** 240 **(E)** 480

6. In an isosceles trapezoid with bases 6 and 16 a circle is inscribed (touching all four sides of the trapezoid). What is the diameter of this circle?

   (A) 9 **(B)** $4\sqrt{5}$ **(C)** $4\sqrt{6}$ **(D)** 10 **(E)** 11

7. When $(a-b+c)^7$ is expanded and simplified how many terms are preceded by a minus sign?

(A) 7 **(B)** 10 **(C)** 11 **(D)** 15 **(E)** 16

8. In two years a son will be one-third as old as his father was 2 years ago. In eighteen years this son will be the same age as his father was 18 years ago. How old is the son now?

(A) 10 **(B)** 12 **(C)** 14 **(D)** 16 **(E)** 18

9. Let m and n be integers satisfying $m^2+n^2=50$. The value of $m+n$ must be

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10. Recall that a Pythagorean triple is a triple (a,b,c) of positive integers satisfying $a^2+b^2=c^2$. Which of the following must be true?

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(D) $\dfrac{60(x-kr)}{h-k-60m}$ **(E)** $\dfrac{60(x-kr)}{60h-60k-m}$

12. The coefficient of x^8 when $(1+x+x^2+x^3+x^4+x^4+x^6+x^7+x^8)^3$ is expanded and similar terms are collected is equal to

(A) 1 **(B)** 8 **(C)** 9 **(D)** 42 **(E)** 45

13. Molly's Motel is adopting a new room key system. The new keys will be square 3×3 cards each with two holes punched in them as in the figure. The two sides (what we would have called the front and back except we cannot tell which is which!) of such a card cannot be distinguished but there is a distinguished edge which is the edge to be inserted in the lock. What is the greatest number of rooms Molly's Motel can have?

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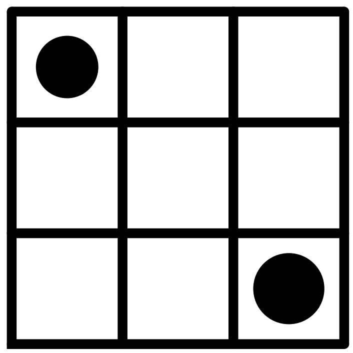

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has the value

(A) 333300 **(B)** 343400 **(C)** 353500 **(D)** 363600 **(E)** 404000

15. Determine the number of ordered pairs (x, y) satisfying the system

$$2x^2 - xy - 6y^2 = 0$$
$$3x^2 - 5xy - 2y^2 = x + 5y + 2$$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

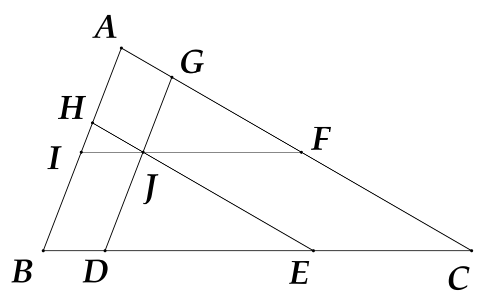
16. In a class of 28 students, 20 take Latin, 14 take Greek, and 10 take Hebrew. If no student takes all three languages and 6 take no language, how many students must be taking both Greek and Hebrew?

(A) 0 **(B)** 1 **(C)** 2
(D) 3 **(E)** cannot be uniquely determined

17. Let a, b, x, and y each be greater than one. If $\log_{ab} x = b$ and $\log_{ab} y = a$, then what is the value of $\log_{xy}(ab)$?

(A) $\dfrac{1}{a+b}$ **(B)** $\dfrac{1}{a} + \dfrac{1}{b}$ **(C)** $a + b$ **(D)** ab **(E)** $\dfrac{a}{b}$

18. In triangle ABC the transversals DG, EH, and FI are concurrent at J, with $DG \parallel AB$, $EH \parallel AC$, and $FI \parallel BC$. If these three transversals have the same length, what is their common length if it is known that $AB = 8$, $BC = 16$, and $CA = 12$?



(A) 91/13 **(B)** 92/13 **(C)** 94/14 **(D)** 95/14 **(E)** 96/13

19. Triangle ABC is inscribed in a circle and $AB = AC = 6$. Point D lies on BC with $AD = 4$. AD is extended through D to E on the circle. Find DE.

(A) 4 **(B)** 5 **(C)** 6
(D) 7 **(E)** cannot be uniquely determined

20. If $r + s = 1$ and $r^4 + s^4 = 4$ find the largest possible value of $r^2 + s^2$.

(A) -2 **(B)** 2 **(C)** 3 **(D)** $-1 + \sqrt{10}$ **(E)** $1 + 2\sqrt{5}$

Spring 2011 McNabb GDCTM Contest
Pre-Calculus

NO Calculators Allowed

1. Hezy leaves home for work at 6:45am. He drives to the Green Line train station 3 miles away at an average speed of 30 mph. After 8 minutes he boards the train for downtown. The train averages 45 mph for its 9 mile journey. After a 7 minute walk Hezy arrives at work. What time does Hezy arrive at work?

   (A) 7:11am **(B)** 7:18am **(C)** 7:21am **(D)** 7:27am **(E)** 7:29am

2. How many arrangements of $REVERE$ are there in which the first R occurs before the first E?

   (A) 12 **(B)** 18 **(C)** 20 **(D)** 24 **(E)** 30

3. In a class, 2/3 of the students have brown eyes and 4/5 of the students have brown hair. If students with brown eyes are twice as likely to have brown hair as students who do not have brown eyes, what fraction of the class has neither brown eyes nor brown hair?

   (A) 1/30 **(B)** 1/15 **(C)** 1/10 **(D)** 2/15 **(E)** 1/5

4. Let a, b, x, and $y > 0$. If $x = by$ and $y = ax$ find the value of $\dfrac{a}{1+a} + \dfrac{b}{1+b}$.

   (A) 1 **(B)** a **(C)** b/a **(D)** 2 **(E)** $1/(a+b)$

5. If n and m are positive integers and $480n = m^2$, what is the smallest possible value of m?

   (A) 90 **(B)** 96 **(C)** 120 **(D)** 240 **(E)** 480

6. In two years a son will be one-third as old as his father was 2 years ago. In eighteen years this son will be the same age as his father was 18 years ago. How old is the son now?

   (A) 10 **(B)** 12 **(C)** 14 **(D)** 16 **(E)** 18

7. Let m and n be integers satisfying $m^2 + n^2 = 50$. The value of $m + n$ must be

(A) -8 **(B)** -5 **(C)** 0
(D) 10 **(E)** cannot be uniquely determined

8. Recall that $i^2 = -1$. Find the value of this complex number

$$\frac{1+i}{1} \cdot \frac{3+i}{2} \cdot \frac{7+i}{5} \cdot \frac{13+i}{10} \cdot \frac{21+i}{17} \cdots \frac{871+i}{842}$$

(A) $30 + 30i$ **(B)** 29 **(C)** $1 + 31i$ **(D)** $30 + i$ **(E)** $1 + 30i$

9. Let w and z be complex conjugate numbers such that w^2/z is a real number. If $|w - z| = 2\sqrt{2}$, what is the value of $|w|^2$?

(A) 8/3 **(B)** 4 **(C)** 5 **(D)** 16/3 **(E)** 6

10. The coefficient of x^8 when $(1 + x + x^2 + x^3 + x^4 + x^4 + x^6 + x^7 + x^8)^3$ is expanded and similar terms are collected is equal to

(A) 1 **(B)** 8 **(C)** 9 **(D)** 42 **(E)** 45

11. The series

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + 100 \cdot 101$$

has the value

(A) 333300 **(B)** 343400 **(C)** 353500 **(D)** 363600 **(E)** 404000

12. In $\triangle ABC$, $AB = \sqrt{2011}$, $\angle C = 120°$, and sides CA and CB are integers. The value of $CA + CB$ could be

(A) 45 **(B)** 46 **(C)** 47 **(D)** 48 **(E)** 49

13. In a class of 28 students, 20 take Latin, 14 take Greek, and 10 take Hebrew. If no student takes all three languages and 6 take no language, how many students must take both Greek and Hebrew?

(A) 0 **(B)** 1 **(C)** 2
(D) 3 **(E)** cannot be uniquely determined

14. In the coordinate plane a laser beam is fired from the origin. After hitting a mirror at $(1,7)$, the beam passes through the point $(15,5)$. The mirror is given by the graph of $ay - bx = c$, where a, b, and c are positive integers with a, b, and c relatively prime. What is the value of $a + b + c$?

(A) 32 **(B)** 33 **(C)** 34 **(D)** 35 **(E)** 36

15. For all positive integers n and m,

$$f(mn+1) = f(n)f(m+1) + f(m)f(n+1) \quad \text{and} \quad f(n) > 0$$

Find the value of $f(11)$.

(A) 1/2 **(B)** 1 **(C)** 3/2 **(D)** 11/2 **(E)** 10

16. Let each of a, b, x, and y be greater than one. If $\log_{ab} x = b$ and $\log_{ab} y = a$, then what is the value of $\log_{xy}(ab)$?

(A) $\dfrac{1}{a+b}$ **(B)** $\dfrac{1}{a} + \dfrac{1}{b}$ **(C)** $a + b$ **(D)** ab **(E)** $\dfrac{a}{b}$

17. The polynomial $p(x) = x^4 - 5x^2 - 6x - 5$ has exactly two real roots, which occur in the form $\dfrac{A \pm \sqrt{B}}{2}$ where A and B are positive integers. Find the value of $A + B$.

(A) 5 **(B)** 12 **(C)** 17 **(D)** 22 **(E)** 24

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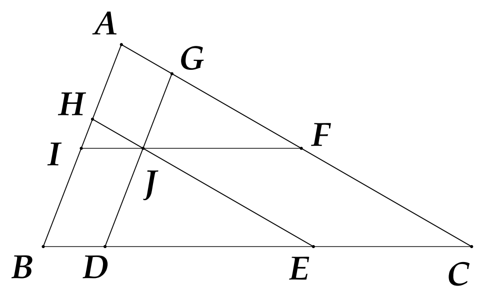

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Spring 2011 McNabb GDCTM Contest
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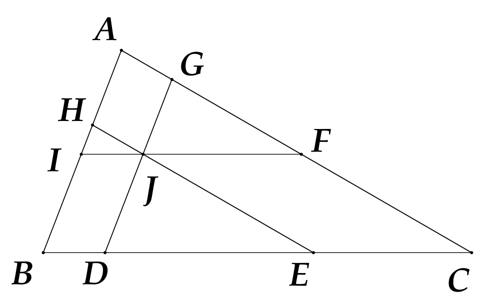

(A) 91/13 **(B)** 93/14 **(C)** 94/14 **(D)** 95/14 **(E)** 96/13

16. Find $\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} e^{k/n}$.

(A) 1 **(B)** 2 **(C)** e **(D)** $e-1$ **(E)** $2e$

17. The line $y = mx$ cuts in half the area of the region bounded by $y = 4x - x^2$ and the x-axis. Find the value of $(4-m)^3$.

(A) 36 **(B)** 32 **(C)** 27 **(D)** 16 **(E)** 8

18. Evaluate $\int_1^{64} \frac{1}{\sqrt{x}+\sqrt[3]{x}}\,dx$.

(A) $11+6\ln(3/2)$ **(B)** $21+6\ln(2/3)$ **(C)** 16
(D) $21-6\ln(2/3)$ **(E)** $11-6\ln(3/2)$

19. Given that for fixed constants A and B

$$\int \sin(2x)\cos(3x)\,dx = A\sin(2x)\sin(3x) + B\cos(2x)\cos(3x) + C$$

find the value of $A+B$.

(A) $1/6$ **(B)** $1/5$ **(C)** $3/5$ **(D)** 1 **(E)** $7/6$

20. Find the value of $\int_0^{\infty} \frac{\ln x}{1+x^2}\,dx$.

(A) $-\infty$ **(B)** $-\pi/4$ **(C)** 0 **(D)** 1 **(E)** $\ln 2$