Spring 2011 McNabb GDCTM Contest
Pre-Calculus

NO Calculators Allowed

1. Hezy leaves home for work at 6:45am. He drives to the Green Line train station 3 miles away at an average speed of 30 mph. After 8 minutes he boards the train for downtown. The train averages 45 mph for its 9 mile journey. After a 7 minute walk Hezy arrives at work. What time does Hezy arrive at work?

   (A) 7:11am **(B)** 7:18am **(C)** 7:21am **(D)** 7:27am **(E)** 7:29am

2. How many arrangements of $REVERE$ are there in which the first R occurs before the first E?

   (A) 12 **(B)** 18 **(C)** 20 **(D)** 24 **(E)** 30

3. In a class, $2/3$ of the students have brown eyes and $4/5$ of the students have brown hair. If students with brown eyes are twice as likely to have brown hair as students who do not have brown eyes, what fraction of the class has neither brown eyes nor brown hair?

   (A) $1/30$ **(B)** $1/15$ **(C)** $1/10$ **(D)** $2/15$ **(E)** $1/5$

4. Let a, b, x, and $y > 0$. If $x = by$ and $y = ax$ find the value of $\dfrac{a}{1+a} + \dfrac{b}{1+b}$.

   (A) 1 **(B)** a **(C)** b/a **(D)** 2 **(E)** $1/(a+b)$

5. If n and m are positive integers and $480n = m^2$, what is the smallest possible value of m?

   (A) 90 **(B)** 96 **(C)** 120 **(D)** 240 **(E)** 480

6. In two years a son will be one-third as old as his father was 2 years ago. In eighteen years this son will be the same age as his father was 18 years ago. How old is the son now?

   (A) 10 **(B)** 12 **(C)** 14 **(D)** 16 **(E)** 18

7. Let m and n be integers satisfying $m^2 + n^2 = 50$. The value of $m + n$ must be

   (A) -8 **(B)** -5 **(C)** 0
   (D) 10 **(E)** cannot be uniquely determined

8. Recall that $i^2 = -1$. Find the value of this complex number

$$\frac{1+i}{1} \cdot \frac{3+i}{2} \cdot \frac{7+i}{5} \cdot \frac{13+i}{10} \cdot \frac{21+i}{17} \cdots \frac{871+i}{842}$$

   (A) $30 + 30i$ **(B)** 29 **(C)** $1 + 31i$ **(D)** $30 + i$ **(E)** $1 + 30i$

9. Let w and z be complex conjugate numbers such that w^2/z is a real number. If $|w - z| = 2\sqrt{2}$, what is the value of $|w|^2$?

   (A) $8/3$ **(B)** 4 **(C)** 5 **(D)** $16/3$ **(E)** 6

10. The coefficient of x^8 when $(1 + x + x^2 + x^3 + x^4 + x^4 + x^6 + x^7 + x^8)^3$ is expanded and similar terms are collected is equal to

    (A) 1 **(B)** 8 **(C)** 9 **(D)** 42 **(E)** 45

11. The series

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + 100 \cdot 101$$

    has the value

    (A) 333300 **(B)** 343400 **(C)** 353500 **(D)** 363600 **(E)** 404000

12. In $\triangle ABC$, $AB = \sqrt{2011}$, $\angle C = 120°$, and sides CA and CB are integers. The value of $CA + CB$ could be

    (A) 45 **(B)** 46 **(C)** 47 **(D)** 48 **(E)** 49

13. In a class of 28 students, 20 take Latin, 14 take Greek, and 10 take Hebrew. If no student takes all three languages and 6 take no language, how many students must take both Greek and Hebrew?

    (A) 0 **(B)** 1 **(C)** 2
    (D) 3 **(E)** cannot be uniquely determined

14. In the coordinate plane a laser beam is fired from the origin. After hitting a mirror at $(1,7)$, the beam passes through the point $(15,5)$. The mirror is given by the graph of $ay - bx = c$, where a, b, and c are positive integers with a, b, and c relatively prime. What is the value of $a+b+c$?

(A) 32 **(B)** 33 **(C)** 34 **(D)** 35 **(E)** 36

15. For all positive integers n and m,

$$f(mn+1) = f(n)f(m+1) + f(m)f(n+1) \quad \text{and} \quad f(n) > 0$$

Find the value of $f(11)$.

(A) $1/2$ **(B)** 1 **(C)** $3/2$ **(D)** $11/2$ **(E)** 10

16. Let each of a, b, x, and y be greater than one. If $\log_{ab} x = b$ and $\log_{ab} y = a$, then what is the value of $\log_{xy}(ab)$?

(A) $\dfrac{1}{a+b}$ **(B)** $\dfrac{1}{a}+\dfrac{1}{b}$ **(C)** $a+b$ **(D)** ab **(E)** $\dfrac{a}{b}$

17. The polynomial $p(x) = x^4 - 5x^2 - 6x - 5$ has exactly two real roots, which occur in the form $\dfrac{A \pm \sqrt{B}}{2}$ where A and B are positive integers. Find the value of $A+B$.

(A) 5 **(B)** 12 **(C)** 17 **(D)** 22 **(E)** 24

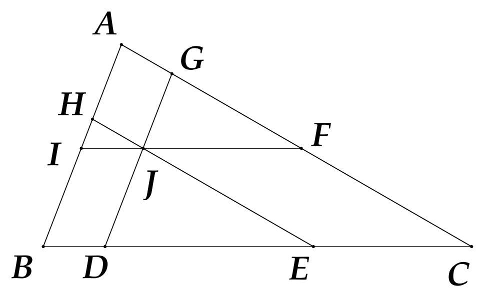
18. Triangle ABC is inscribed in a circle and $AB = AC = 6$. Point D lies on BC with $AD = 4$. AD is extended through D to E on the circle. Find DE.

(A) 4 **(B)** 5 **(C)** 6
(D) 7 **(E)** cannot be uniquely determined

19. If $r+s=1$ and $r^4+s^4=4$ find the largest possible value of r^2+s^2.

(A) -2 **(B)** 2 **(C)** 3 **(D)** $-1+\sqrt{10}$ **(E)** $1+2\sqrt{5}$

20. In triangle ABC the transversals DG, EH, and FI are concurrent at J, with $DG \parallel AB$, $EH \parallel AC$, and $FI \parallel BC$. If these three transversals have the same length, what is their common length if it is known that $AB = 8$, $BC = 16$, and $CA = 12$?



(A) 91/13 **(B)** 92/13 **(C)** 94/14 **(D)** 95/14 **(E)** 96/13