Spring 2011 McNabb GDCTM Contest
Algebra II

NO Calculators Allowed

1. Hezy leaves home for work at 6:45am. He drives to the Green Line train station 3 miles away at an average speed of 30 mph. After 8 minutes he boards the train for downtown. The train averages 45 mph for its 9 mile journey. After a 7 minute walk Hezy arrives at work. What time does Hezy arrive at work?

   (A) 7:11am **(B)** 7:18am **(C)** 7:21am **(D)** 7:27am **(E)** 7:29am

2. How many arrangements of $REVERE$ are there in which the first R occurs before the first E?

   (A) 12 **(B)** 18 **(C)** 20 **(D)** 24 **(E)** 30

3. In a class, $2/3$ of the students have brown eyes and $4/5$ of the students have brown hair. If students with brown eyes are twice as likely to have brown hair as students who do not have brown eyes, what fraction of the class has neither brown eyes nor brown hair?

   (A) $1/30$ **(B)** $1/15$ **(C)** $1/10$ **(D)** $2/15$ **(E)** $1/5$

4. Let a, b, x, and $y > 0$. If $x = by$ and $y = ax$ find the value of $\dfrac{a}{1+a} + \dfrac{b}{1+b}$.

   (A) 1 **(B)** a **(C)** b/a **(D)** 2 **(E)** $1/(a+b)$

5. If n and m are positive integers and $480n = m^2$, what is the smallest possible value of m?

   (A) 90 **(B)** 96 **(C)** 120 **(D)** 240 **(E)** 480

6. In an isosceles trapezoid with bases 6 and 16 a circle is inscribed (touching all four sides of the trapezoid). What is the diameter of this circle?

   (A) 9 **(B)** $4\sqrt{5}$ **(C)** $4\sqrt{6}$ **(D)** 10 **(E)** 11

7. When $(a-b+c)^7$ is expanded and simplified how many terms are preceded by a minus sign?

   (A) 7 **(B)** 10 **(C)** 11 **(D)** 15 **(E)** 16

8. In two years a son will be one-third as old as his father was 2 years ago. In eighteen years this son will be the same age as his father was 18 years ago. How old is the son now?

   (A) 10 **(B)** 12 **(C)** 14 **(D)** 16 **(E)** 18

9. Let m and n be integers satisfying $m^2+n^2=50$. The value of $m+n$ must be

   (A) -8 **(B)** -5 **(C)** 0
   (D) 10 **(E)** cannot be uniquely determined

10. Recall that a Pythagorean triple is a triple (a,b,c) of positive integers satisfying $a^2+b^2=c^2$. Which of the following must be true?

    (I.) At least one of a, b, and c must be odd

    (II.) At least one of a, b, and c must be even

    (III.) For at least one Pythagorean triple, $a=b$.

    (A) I only **(B)** II only **(C)** I and II only
    (D) II and III only **(E)** none of them

11. A train having to journey x miles in h hours, ran for k hours at a rate of r miles per hour, then stopped for m minutes. How fast must it go (in mph) on the rest of its journey to arrive on time?

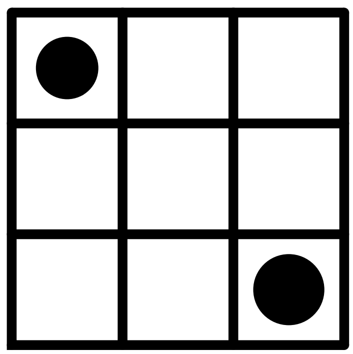
    (A) $\dfrac{x-kr}{h-k-m}$ **(B)** $\dfrac{x-kr}{60h-60k-m}$ **(C)** $\dfrac{60(x-kr)}{h-k-m}$
    (D) $\dfrac{60(x-kr)}{h-k-60m}$ **(E)** $\dfrac{60(x-kr)}{60h-60k-m}$

12. The coefficient of x^8 when $(1+x+x^2+x^3+x^4+x^4+x^6+x^7+x^8)^3$ is expanded and similar terms are collected is equal to

    (A) 1 **(B)** 8 **(C)** 9 **(D)** 42 **(E)** 45

13. Molly's Motel is adopting a new room key system. The new keys will be square 3×3 cards each with two holes punched in them as in the figure. The two sides (what we would have called the front and back except we cannot tell which is which!) of such a card cannot be distinguished but there is a distinguished edge which is the edge to be inserted in the lock. What is the greatest number of rooms Molly's Motel can have?

insert this edge



(A) 12 **(B)** 18 **(C)** 21 **(D)** 24 **(E)** 36

14. The series

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + 100 \cdot 101$$

has the value

(A) 333300 **(B)** 343400 **(C)** 353500 **(D)** 363600 **(E)** 404000

15. Determine the number of ordered pairs (x, y) satisfying the system

$$2x^2 - xy - 6y^2 = 0$$
$$3x^2 - 5xy - 2y^2 = x + 5y + 2$$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

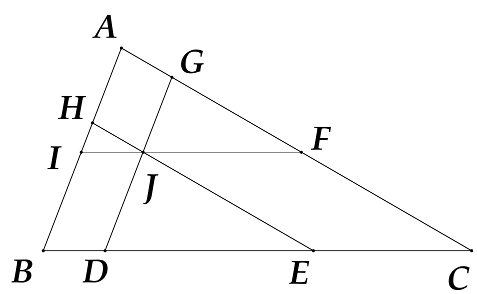
16. In a class of 28 students, 20 take Latin, 14 take Greek, and 10 take Hebrew. If no student takes all three languages and 6 take no language, how many students must be taking both Greek and Hebrew?

(A) 0 **(B)** 1 **(C)** 2
(D) 3 **(E)** cannot be uniquely determined

17. Let a, b, x, and y each be greater than one. If $\log_{ab} x = b$ and $\log_{ab} y = a$, then what is the value of $\log_{xy}(ab)$?

(A) $\dfrac{1}{a+b}$ **(B)** $\dfrac{1}{a} + \dfrac{1}{b}$ **(C)** $a+b$ **(D)** ab **(E)** $\dfrac{a}{b}$

18. In triangle ABC the transversals DG, EH, and FI are concurrent at J, with $DG \parallel AB$, $EH \parallel AC$, and $FI \parallel BC$. If these three transversals have the same length, what is their common length if it is known that $AB = 8$, $BC = 16$, and $CA = 12$?



(A) 91/13 **(B)** 92/13 **(C)** 94/14 **(D)** 95/14 **(E)** 96/13

19. Triangle ABC is inscribed in a circle and $AB = AC = 6$. Point D lies on BC with $AD = 4$. AD is extended through D to E on the circle. Find DE.

(A) 4 **(B)** 5 **(C)** 6
(D) 7 **(E)** cannot be uniquely determined

20. If $r + s = 1$ and $r^4 + s^4 = 4$ find the largest possible value of $r^2 + s^2$.

(A) -2 **(B)** 2 **(C)** 3 **(D)** $-1 + \sqrt{10}$ **(E)** $1 + 2\sqrt{5}$