Spring 2011 McNabb GDCTM Contest
Algebra I

NO Calculators Allowed

1. Hezy leaves home for work at 6:45am. He drives to the Green Line train station 3 miles away at an average speed of 30 mph. After 8 minutes he boards the train for downtown. The train averages 45 mph for its 9 mile journey. After a 7 minute walk Hezy arrives at work. What time does Hezy arrive at work?

   (A) 7:11am **(B)** 7:18am **(C)** 7:21am **(D)** 7:27am **(E)** 7:29am

2. How many arrangements of $REVERE$ are there in which the first R occurs before the first E?

   (A) 12 **(B)** 18 **(C)** 20 **(D)** 24 **(E)** 30

3. If $a▲b = b(a+1)$ what is the value of $(a▲1)▲(1▲a)$?

   (A) $2a^2+4a$ **(B)** $2a^2+3a+1$ **(C)** a^2+3a+2
   (D) a^2+3a **(E)** $6a$

4. Suppose that \$600 is divided into two parts in the ratio of 2 : 3. The larger of these parts is then further subdivided into two parts in the ratio of 3 : 2. The smallest of these now three parts is

   (A) \$96 **(B)** \$144 **(C)** \$192 **(D)** \$216 **(E)** \$240

5. A fifth number n is added to the set $\{3,6,9,10\}$ to form a new set $\{3,6,9,10,n\}$. For how many values of n is the mean of this new set equal to its own median?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** more than 4

6. How many different rectangular prisms can be made using exactly 48 unit cubes?

   (A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

7. In a class, $2/3$ of the students have brown eyes and $4/5$ of the students have brown hair. If students with brown eyes are twice as likely to have brown hair as students who do not have brown eyes, what fraction of the class has neither brown eyes nor brown hair?

   (A) $1/30$ **(B)** $1/15$ **(C)** $1/10$ **(D)** $2/15$ **(E)** $1/5$

8. When three different numbers from the set $\{-7, -2, -1, 0, 4, 5\}$ are multiplied together the smallest possible product is

   (A) -343 **(B)** -175 **(C)** -140 **(D)** -14 **(E)** 0

9. Out of a sphere of clay with diameter 12, Marty fashions two spheres of radius 3 and 5 respectively. Using all of the remaining clay Jennifer fashions a sphere. What is the diameter of Jennifer's sphere?

   (A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

10. The product $60 \times 60 \times 24 \times 7$ equals

    (A) the number of minutes in seven weeks
    (B) the number of hours in sixty days
    (C) the number of seconds in seven hours
    (D) the number of seconds in one week
    (E) the number of minutes in twenty-four weeks

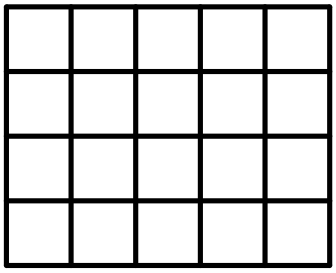
11. Let a, b, x, and $y > 0$. If $x = by$ and $y = ax$ find the value of $\dfrac{a}{1+a} + \dfrac{b}{1+b}$.

    (A) 1 **(B)** a **(C)** b/a **(D)** 2 **(E)** $1/(a+b)$

12. If n and m are positive integers and $480n = m^2$, what is the smallest possible value of m?

    (A) 90 **(B)** 96 **(C)** 120 **(D)** 240 **(E)** 480

13. How many rectangles are in this figure?



(A) 20 **(B)** 75 **(C)** 150 **(D)** 300 **(E)** 600

14. The sum of two positive numbers is S and their positive difference is $1/m$th of the smaller number. What is the value of the larger number?

(A) $\frac{mS}{2m+1}$ **(B)** $\frac{(m-1)S}{2m}$ **(C)** $\frac{m^2S}{2m-1}$ **(D)** $\frac{2mS}{m+1}$ **(E)** $\frac{(m+1)S}{2m+1}$

15. A regular 52 card deck is well shuffled. What is the probability that both the top and bottom cards are aces?

(A) $1/26$ **(B)** $1/52$ **(C)** $3/221$ **(D)** $2/221$ **(E)** $1/221$

16. In two years a son will be one-third as old as his father was 2 years ago. In eighteen years this son will be the same age as his father was 18 years ago. How old is the son now?

(A) 10 **(B)** 12 **(C)** 14 **(D)** 16 **(E)** 18

17. Let $f(x)$ be a linear function satisfying $f(0) = 0$. If both $f(a+b) = 7$ and $f(a-2b) = 3$, then the value of $f(a+7b)$ must be

(A) 9 **(B)** 11 **(C)** 13
(D) 15 **(E)** cannot be uniquely determined

18. Recall that a Pythagorean triple is a triple (a, b, c) of positive integers satisfying $a^2 + b^2 = c^2$. Which of the following must be true?

(I.) At least one of a, b, and c must be odd

(II.) At least one of a, b, and c must be even

(III.) For at least one Pythagorean triple, $a = b$.

(A) I only **(B)** II only **(C)** I and II only
(D) II and III only **(E)** none of them

19. A train having to journey x miles in h hours, ran for k hours at a rate of r miles per hour, then stopped for m minutes. How fast must it go (in mph) on the rest of its journey to arrive on time?

(A) $\dfrac{x - kr}{h - k - m}$ **(B)** $\dfrac{x - kr}{60h - 60k - m}$ **(C)** $\dfrac{60(x - kr)}{h - k - m}$
(D) $\dfrac{60(x - kr)}{h - k - 60m}$ **(E)** $\dfrac{60(x - kr)}{60h - 60k - m}$

20. The image of the line $y = 4x - 6$ under reflection across the line $y = -x$ is the line

(A) $y = \frac{1}{4}x - \frac{3}{2}$ **(B)** $y = -\frac{1}{4}x + \frac{3}{2}$ **(C)** $y = \frac{1}{4}x - \frac{4}{3}$ **(D)** $y = \frac{1}{4}x - 1$
(E) $y = \frac{1}{4}x + \frac{2}{3}$

21. Let m and n be integers satisfying $m^2 + n^2 = 50$. The value of $m + n$ must be

(A) -8 **(B)** -5 **(C)** 0
(D) 10 **(E)** cannot be uniquely determined

22. In a class of 28 students, 20 take Latin, 14 take Greek, and 10 take Hebrew. If no student takes all three languages and 6 take no language, how many students must be taking both Greek and Hebrew?

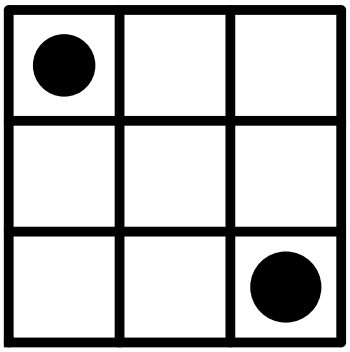
(A) cannot be uniquely determined **(B)** 0 **(C)** 1
(D) 2 **(E)** 3

23. The area of rectangle $ABCD$ is 40. Point P is on AB so that $BP = 3$. Point R is on AD so that $DR = 2$. Given that $APQR$ is a rectangle with area of 15, find the average of the two possible values for the length of AP.

(A) $19/4$ **(B)** $21/4$ **(C)** $19/2$ **(D)** $21/2$ **(E)** 5

24. Molly's Motel is adopting a new room key system. The new keys will be square 3×3 cards each with two holes punched in them as in the figure. The two sides (what we would have called the front and back except we cannot tell which is which!) of such a card cannot be distinguished but there is a distinguished edge which is the edge to be inserted in the lock. What is the greatest number of rooms Molly's Motel can have?

insert this edge



(A) 12 **(B)** 18 **(C)** 21 **(D)** 24 **(E)** 36

25. What is the remainder when $x^{14} + x^{11} + x^8 + x^5 + x^3 + x^2 + x + 1$ is divided by $x^2 - x + 1$?

(A) 3 **(B)** $2x$ **(C)** $4x + 1$ **(D)** $2x - 1$ **(E)** $-x + 4$